Viscosities of Quark Gluon Plasma with Nonzero Baryon Number

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QCD Phase Diagram

- Quark gluon plasma is an ideal fluid
- The system is in state of equilibrium
\[ T^{\mu\nu} = -pg^{\mu\nu} + (p + \varepsilon)u^\mu u^\nu \]

Stress energy tensor for ideal fluid

**Conserved Quantities**

**Energy conservation**

\[ \partial_\mu T^{\mu\nu} = 0 \]

1) \[ \partial_t \varepsilon + (p + \varepsilon)(\nabla \cdot \vec{u}) = 0 \]

**Baryon number conservation**

\[ \partial_\mu (nu^\mu) = 0 \]

2) \[ \partial_t n + n(\nabla \cdot \vec{u}) = 0 \]
\[ T^{0i}(x) = 0 \quad \iff \quad \text{local fluid rest frame at } x \]

\[
 u_i \equiv \frac{\langle T^{0i} \rangle}{\langle \varepsilon + P \rangle} \quad \quad \quad 0 \equiv \delta T^{00}
\]

**Constitutive Relations**

\[
\langle T^{ij} \rangle = \delta^{ij} \langle P \rangle - \eta \left[ \nabla^i u^j + \nabla^j u^i - \frac{2}{3} \delta^{ij} \nabla^l u_l \right] - \zeta \delta^{ij} \nabla^l u_l
\]

\[
\zeta = \zeta(T, \mu) \quad \quad \quad \eta = \eta(T, \mu)
\]
Consider that there are two plates and if top plate moves with \( u \) velocity, particles transfer momentum to each other and the bottom feels the move.

- **Kinetic Theory**
  Equilibrium distribution functions for quark, antiquark and gluon in Quark gluon plasma

\[
f_{eq}^{q\bar{q}}(x, \vec{p}, t) = \left[ \exp \beta(t) \gamma \left[ E_p(t) - \vec{p} \cdot \vec{u}(x) \pm \mu(t) \right] + 1 \right]^{-1}
\]

\[
f_{eq}^{\text{glue}}(x, \vec{p}, t) = \left[ \exp \beta(t) \gamma \left[ E_p(t) - \vec{p} \cdot \vec{u}(x) \right] - 1 \right]^{-1}
\]
Boltzmann Equation

\[ \frac{df(x, t)}{dt} = C[f] \]

Collision term

\[ \frac{\partial f_{eq}}{\partial t} + V_i \frac{\partial f_{eq}}{\partial x} = C[f_{eq}](p) = 0 \]

If use equilibrium distribution function in collision term, Right hand side of equation will be zero

Linearized of the boltzmann equation must be

\[ \frac{df_{eq}(x, t)}{dt} = \frac{\partial f_{eq}}{\partial t} + V_i \frac{\partial f_{eq}}{\partial x} = C[f_1](p) \]
Collision integral

\[
C[f_1](p) = \frac{1}{2} \int |M(p, k, p', k')|^2 (2\pi)^4 \delta^4(p + k - p' - k') \\
\times f_{eq}(p)f_{eq}(k)[1 \pm f_{eq}(p')][1 \pm f_{eq}(k')] \\
\times [f_1(p) + f_1(k) - f_1(p') - f_1(k')]
\]

Linearized Boltzmann Equation

\[
LHS = \beta f_{eq}[1 \pm f_{eq}] q^\alpha I_{ij}(p) X_{ij}(x)
\]
\[ X_{ij}(x) \equiv (\nabla_i u_j - \nabla_i u_j - \frac{2}{3} \delta_{ij} \nabla \cdot \mathbf{u}) \]

\[ I_{ij}(p) \equiv \begin{cases} 
\frac{3}{2} (\hat{p}_i \hat{p}_j - \frac{1}{3} \delta_{ij}) & \text{, shear viscosity} \\
1 & \text{, bulk viscosity} 
\end{cases} \]

**Source term**

\[ S = C\chi \]
For Leading Logarithmic order

In the limit $g$ goes to zero, a logarithmic expansion can be made

$$C \sim g^4 \quad \longrightarrow \quad C \sim g^4 \log \left( \frac{1}{g} \right)$$

Shear viscosity

$$\eta = \langle \chi, C\chi \rangle$$

$$= \frac{\beta^3}{8} \int |M(p,k,p',k')|^2 (2\pi)^4 \delta^4(p+k-p'-k')$$

$$\times f_{eq}(p)f_{eq}(k)[1 \pm f_{eq}(p')][1 \pm f_{eq}(k')]$$

$$\times [\chi(p) + \chi(k) - \chi(p') - \chi(k')]^2$$

$$\eta \approx \frac{C_1 \left( \frac{\mu}{T} \right) T^3}{g^4 \log \left( \frac{1}{g} \right)}$$
LL order solutions

\[ \eta \approx \eta_{LL} + \eta_{NLL} \]

Next to LL order solutions

\[ \eta_{NLL} = [\eta_{LL} - (\chi_{LL}, C \chi_{LL})] \]

\( (\chi_{LL}, C \chi_{LL}) \)

\[ = \frac{\beta^3}{8} \int |M(p, k, p', k')|^2 (2\pi)^4 \delta^4(p + k - p' - k') \]

\[ \times f_{eq}(p)f_{eq}(k)[1 \pm f_{eq}(p')] [1 \pm f_{eq}(k')] \]

\[ \times [\chi_{LL}(p) + \chi_{LL}(k) - \chi_{LL}(p') - \chi_{LL}(k')]^2 \]

\[ \eta = \frac{T^3 C_1(\mu/T)}{g^4 \log \left[ \frac{C_2(\mu/T)}{g} \right]} \]
The calculations of shear viscosity were done by J.-Wei Chen, Y.-Fu Liu, Y.-Kun Song, and Q. Wang [3]
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\[ \frac{\mu}{T} = 0.9 \]

Solutions of quark and antiquark are separated
Bulk Viscosity

If an isotropic system is expanding or compressing

\[ \zeta \propto T^3 g^4 \]

Comes from Feynman diagrams

In QCD

\[ P(\mu, T) = (s_1 + s_2 g^2)T^4 \times f(\mu/T) \]
Using conservation equations

\[ \partial_{\mu} T^{\mu\nu} = 0 \quad \partial_{\mu} (nu^\mu) = 0 \]

\[ \beta = \frac{1}{T} \]

\[ \frac{\partial \beta}{\partial t} = \left[ (P + \varepsilon) \frac{\partial n}{\partial \mu} - n \frac{\partial \varepsilon}{\partial \mu} \right] \left( \frac{\partial \varepsilon}{\partial \mu} \frac{\partial n}{\partial \beta} - \frac{\partial \varepsilon}{\partial \beta} \frac{\partial n}{\partial \mu} \right)^{-1} (\nabla \cdot \vec{u}) \]

\[ \frac{\partial \mu}{\partial t} = - \left[ (P + \varepsilon) \frac{\partial n}{\partial \beta} - n \frac{\partial \varepsilon}{\partial \beta} \right] \left( \frac{\partial \varepsilon}{\partial \mu} \frac{\partial n}{\partial \beta} - \frac{\partial \varepsilon}{\partial \beta} \frac{\partial n}{\partial \mu} \right)^{-1} (\nabla \cdot \vec{u}) \]

If \( \varepsilon = 3P \) \[ \rightarrow \quad S = 0 \quad \text{Source term of bulk viscosity} \]

**Thermodynamic relations**

\[ \varepsilon = -P + T \frac{\partial P}{\partial T} + \mu \frac{\partial P}{\partial \mu} \]
\[ \beta(g^2) = T \frac{\partial g^2}{\partial T} + \mu \frac{\partial g^2}{\partial \mu} \]

g^2 must flow for \( \zeta \) not to be zero

\[ \zeta \propto [\beta(g^2)]^2 \]

- It is known what bulk viscosity should be and it can be calculated
- It can be calculate next to leading logarithmic order solutions of shear viscosity

References

For source term of quark and antiquark

\[
\frac{\partial f_{eq}^{q\bar{q}}}{\partial t} = (\vec{\nabla} \cdot \vec{u}) \left\{ \left( \frac{\partial \varepsilon}{\partial \mu} \frac{\partial n}{\partial \beta} - \frac{\partial \varepsilon}{\partial \beta} \frac{\partial n}{\partial \mu} \right)^{-1} \left\{ \left( P + \varepsilon \right) \frac{\partial n}{\partial \mu} - n \frac{\partial \varepsilon}{\partial \mu} \right\} \times \left[ \frac{\partial (E_p \beta)}{\partial \beta} \mp \mu \right] - \left[ (P + \varepsilon) \frac{\partial n}{\partial \beta} - n \frac{\partial \varepsilon}{\partial \beta} \right] \times \left[ \frac{\partial (E_p \beta)}{\partial \mu} \mp \beta \right] \right\} - \frac{\beta}{3} \vec{V} \cdot \vec{u} \right\}
\]
For source term of gluon

\[
\frac{\partial f_{eq}^{\text{glue}}}{\partial t} = (\vec{\nabla}.\vec{u}) \left\{ \left( \frac{\partial \varepsilon}{\partial \mu} \frac{\partial n}{\partial \beta} - \frac{\partial \varepsilon}{\partial \beta} \frac{\partial n}{\partial \mu} \right)^{-1} \left\{ \left[ (P + \varepsilon) \frac{\partial n}{\partial \mu} - n \frac{\partial \varepsilon}{\partial \mu} \right] \times \left( \frac{\partial (E_p \beta)}{\partial \beta} \right) \right\} - \left[ (P + \varepsilon) \frac{\partial n}{\partial \beta} - n \frac{\partial \varepsilon}{\partial \beta} \right] \times \left( \frac{\partial (E_p \beta)}{\partial \mu} \right) \right\} \right\} - \frac{\beta}{3} (\vec{V}.\vec{u})
\]