Stability of the Standard Model ground state
A precision analysis

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The Standard Model: $SU(3)_c \times SU(2)_L \times U(1)_Y$

- gauge couplings: QCD: $g_s$ Electroweak: $g_2$, $g_1$
- Yukawa couplings and Higgs self-interaction:

### Spontaneous Symmetry Breaking

**Classical Higgs potential:**

$$V(|\Phi|) = m^2 \Phi^* \Phi + \lambda (\Phi^* \Phi)^2$$

| $\Phi_c$ | $\langle 0 | \Phi(x) | 0 \rangle$ |
|---|---|
| $|\Phi_c|$ | $\frac{v}{\sqrt{2}} \neq 0$ |
| $v$ | $\approx 246.2 \text{ GeV}$ |
Stability of the ground state

QFT: Radiative corrections → $V_{\text{eff}}(\lambda(\Lambda), g_i(\Lambda), y_i(\Lambda), \ldots)[\Phi(\Lambda)]$  [Coleman, Weinberg]

($\Lambda$: scale up to which the SM is valid, starting scale for running e.g. $\mu_0 = M_t$)

For $\Phi \sim \Lambda \gg v$: $V_{\text{eff}}[\Phi] \approx \lambda(\Lambda)\Phi(\Lambda)^4$  [Altarelli, Isidori; Ford, Jack, Jones]

Stability of SM vacuum $\Leftrightarrow \lambda(\Lambda) > 0$  [Cabibbo; Sher; Lindner; Ford]
Evolution of $\lambda$ up to $\Lambda \sim $ $M_{\text{Planck}}$

$\lambda(\mu)$ for different values of $M_H$

- $M_H = 170$ GeV
- $M_H = 135$ GeV
- $M_H = 125$ GeV
- $M_H = 115$ GeV
- $M_H = 80$ GeV

$\log_{10}[\mu/\text{GeV}]$
Evolution of couplings $X \in \{\lambda, g_1, g_2, g_s, y_t\}$

$\beta$-functions:

$$\mu^2 \frac{d}{d\mu^2} X(\mu^2) = \beta_X [\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2), \ldots]$$

$\mu$: energy scale of a given physical process

$\Rightarrow$ Coupled system of differential equations with initial conditions

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Three-loop $\beta$-functions in the SM

- for gauge couplings $g_1, g_2, g_s$: [Mihaila, Salomon, Steinhauser (2012); Bednyakov, Pikelner, Velizhanin (2012)]

- for Yukawa couplings $y_t, y_b, y_\tau$, etc.: [Chetyrkin, MZ (2012); Bednyakov, Pikelner, Velizhanin (2013)]

- for the Higgs self-coupling $\lambda$ (and the mass parameter $m^2$): [Chetyrkin, MZ (2012 and 2013); Bednyakov, Pikelner, Velizhanin (2013)]
Calculation of $\beta_\lambda(\lambda, y_t, g_s, g_2, g_1)$

$= \frac{y_t^4}{(16\pi^2)} + \frac{\lambda^2}{(16\pi^2)} + \cdots + \delta Z_\lambda$

More loops $\Rightarrow$ more precision

$\propto \frac{y_t^4 \lambda}{(16\pi^2)^2}$
$\propto \frac{y_t^4 g_s^2}{(16\pi^2)^2}$
$\propto \frac{y_t^4 g_s^4}{(16\pi^2)^3}$

$\propto \frac{y_t^8}{(16\pi^2)^3}$
$\propto \frac{y_t^6 g_s^2}{(16\pi^2)^3}$
$\propto \frac{y_t^4 g_s^4}{(16\pi^2)^3}$

Challenges:
- $O(10^6)$ diagrams at 3 loops
- Treatment of $\gamma_5$ in $D = 4 - 2\varepsilon$ ['t Hooft, Veltman]
- IR divergencies [Chetyrkin, Misiak, Münz]
Results:
\[ \mu^2 \frac{d}{d\mu^2} \lambda(\mu) = \beta_\lambda = \sum_{n=1}^{\infty} \frac{1}{(16\pi^2)^n} \beta_\lambda^{(n)} \] (in the \( \overline{\text{MS}} \)-scheme)

\[ \beta_\lambda^{(1)} = -y_t^4 3 + y_t^2 \lambda 6 - \lambda g_2^2 \frac{9}{2} + \lambda^2 12 + g_2^4 \frac{9}{16} - \lambda g_1^2 \frac{3}{2} + g_1^2 g_2^2 \frac{3}{8} + g_1^4 \frac{3}{16} + \lambda y_t^6 + \lambda y_t^2 2 - y_t^4 3 - y_t^4 \]

\[ \beta_\lambda^{(2)} = -g_2^2 y_t^4 16 + y_t^6 15 + g_2^2 y_t^2 \lambda 40 - y_t^2 \lambda^2 72 + y_t^4 \lambda g_2^2 \frac{45}{4} + + \lambda^2 g_2^4 54 - \lambda^3 156 + \ldots \]

\[ \beta_\lambda^{(3)} = g_2^2 y_t^6 (-38 + 240 \zeta_3) + y_t^8 \left(-\frac{1599}{8} - 36 \zeta_3\right) + g_2^4 y_t^4 \left(-\frac{626}{3} + 32 \zeta_3 + 40 N_g\right) \]

\[ + g_2^2 y_t^4 \lambda (895 - 1296 \zeta_3) + g_2^4 y_t^2 \lambda \left(\frac{1820}{3} - 48 \zeta_3 - 64 N_g\right) + y_t^4 \lambda^2 \left(\frac{1719}{2} + 756 \zeta_3\right) \]

\[ + y_t^6 g_2^2 \left(\frac{3411}{32} - 27 \zeta_3\right) + y_t^6 \lambda \left(\frac{117}{8} - 198 \zeta_3\right) + \ldots \]

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Starting values for couplings
[Sirlin, Zucchini; Hempfling, Kniehl; Jegerlehner et al; Bezrukov et al; Buttazzo et al]

From experimental data: \( \overline{\text{MS}} \) parameters:

\[ M_t \approx 173.34 \ \text{GeV} \quad \Rightarrow \quad g_s(M_t) \approx 1.16 \quad y_t(M_t) \approx 0.94 \]

\[ M_H \approx 125.9 \ \text{GeV} \quad \Rightarrow \quad g_2(M_t) \approx 0.65 \quad g_1(M_t) \approx 0.36 \]

\[ \alpha_s \approx 0.1184 \]

\[ \lambda(M_t) \approx 0.13 \]
$M_H = 125.9$ GeV, $M_t = 173.34$ GeV, $\alpha_s = 0.1184$
$M_H = 125.9$ GeV, $M_t = 173.34$ GeV, $\alpha_s = 0.1184$

- 3 loop
- 2 loop
- $\alpha_s = 0.1184 + 0.0007$
- $\alpha_s = 0.1184 - 0.0007$
- $M_H = 125.9 + 0.4$ GeV
- $M_H = 125.9 - 0.4$ GeV
- $M_t = 173.34 - 0.76$ GeV
- $M_t = 173.34 + 0.76$ GeV

Graph showing the relationship between $\lambda(\mu)$ and $\log_{10}[\mu/\text{GeV}]$. The graph compares different values of $\alpha_s$ and $M_H$, $M_t$.
Summary

- Stability of SM vacuum $\leftrightarrow \lambda > 0$

- 3 loop $\beta_{\lambda}$ effect smaller than experimental uncertainty.

- 3 loop $\beta_{\lambda}$ result improves stability.

- Vacuum state at the electroweak scale $v$ seems not to be the absolute minimum of the effective Higgs potential for the SM up to $M_{\text{Planck}}$.

- However, there are large experimental uncertainties! Largest uncertainty: $M_t \Rightarrow$ Good reason for a linear $e^+e^-$ collider.

$\Rightarrow$ Question of vacuum stability remains open.