Event-by-event hydrodynamical description of QCD matter

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Outline

1. sQGP and the hydrodynamical approach
2. N-pole asymmetries in the description
3. The elliptical Buda-Lund model and its properties
4. General asymmetries in the model
5. Observables from the generalized model
sQGP

- sQGP discovered at RHIC and also created at LHC
- Almost perfect fluid, expanding hydrodynamical system
- Hadrons created at the freeze-out, leptons, photons created previous the freeze-out too
Perfect fluid hydrodynamics

- Hydro solutions or models
- Relativistic, exact, analytic solution:
  - Famous solution: Landau-Khalatnikov, Hwa-Bjorken
  - There is many new solutions
  - Geometry?
- The most basic concept: spherical symmetry
- Non-central collisions → assuming elliptical asymmetry
- More precise description: higher order asymmetries including!
- Generalize the space-time and the velocity field distribution too!
The elliptical Buda-Lund model


- Final state parametrization with source function:
  \[ S(x, p) = \frac{g}{(2\pi)^3} \frac{p^\mu d^4\Sigma_\mu(x)}{B(x, p) + s_q} \]

  \[ p^\mu d^4\Sigma_\mu(x) = p_\mu u^\mu \delta(\tau - \tau_0)d^4x \] the Cooper-Frye factor, assuming instant freeze-out. \( B(x, p) \) is the Boltzmann-factor.

- Scaling variable:
  \[ s = \frac{r_x^2}{2X^2} + \frac{r_y^2}{2Y^2} + \frac{r_z^2}{2Z^2} \]

- Thermodynamical quantities depend only on \( s \) not on the coordinates

- Derived the velocity field from a potential: \( u_\mu = \gamma (1, \partial_x \Phi, \partial_y \Phi, \partial_z \Phi) \)

  \[ \Phi = \left( \frac{\dot{X}}{2X} + \frac{\dot{Y}}{2Y} + \frac{\dot{Z}}{2Z} \right) \]
Generalization

Spatial distribution (with $\epsilon_n$ asymmetry parameter):

- Elliptical symmetry: $s = \frac{r^2}{R^2} (1 + \epsilon_2 \cos(2\varphi)) + \frac{r_z^2}{Z^2}$
- Triangular symmetry: $s = \frac{r^2}{R^2} (1 + \epsilon_3 \cos(3\varphi)) + \frac{r_z^2}{Z^2}$
- Generally:

$$s = \frac{r^2}{R^2} \left( 1 + \sum_{n=2}^{N} \epsilon_n \cos(n\varphi) \right) + \frac{r_z^2}{Z^2}$$

This $s$ can be used in a hydro solution: Csanád, Szabó PhysRevC.90.054911

The generalized potential of velocity field (with $\chi_n$ asymmetry parameter):

- Elliptical symmetry: $\Phi = \frac{r^2}{2H} (1 + \chi_2 \cos(2\varphi)) + \frac{r_z^2}{2H_z}$
- Triangular symmetry: $\Phi = \frac{r^2}{2H} (1 + \chi_3 \cos(3\varphi)) + \frac{r_z^2}{2H_z}$
- Generally:

$$\Phi = \frac{r^2}{2H} \left( 1 + \sum_{n=2}^{N} \chi_n \cos(n\varphi) \right) + \frac{r_z^2}{2H_z}$$
Observables from the new model

- Invariant momentum distribution: \( N_1(p) = \int S(x, p) d^4x \)

- Flows: \( N_1(p) = N_1(p_t) \left( 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n \alpha) \right) \)
  where the flow \( v_n(p_t) = \langle \cos(n \alpha) \rangle \)

- Bose-Einstein correlations: The correlation function is the Fourier transformation of the source function: \( C(q) = 1 + | \int S(r) \exp(iqr) dr |^2 \)

The asymmetries is measured in the corresponding event plane
- Elliptical asymmetry \( \rightarrow 2^{\text{nd}} \) order event plane
- Triangular asymmetry \( \rightarrow 3^{\text{rd}} \) order event plane

- No interplay among the asymmetries!
Flo ws from the mo del

- Elliptic ($v_2$) and triangular ($v_3$) flows can be derived from the model
- Mixing of the parameters: the spatial distribution asymmetry ($\epsilon_{2,3}$) and the velocity field asymmetry ($\chi_{2,3}$) form the flows together
Azimuthally sensitive HBT radii

Useful to describe the geometry of the source

- The correlation function is the Fourier transformation of the source
- Elliptical case: both of it is Gaussian but with inverse width

\[ S(r) \sim e^{-\frac{r_x^2}{2R_x^2} - \frac{r_y^2}{2R_y^2} - \frac{r_z^2}{2R_z^2}} \rightarrow C(k) = 1 + e^{-k_x^2R_x^2 - k_y^2R_y^2 - k_z^2R_z^2} \]

- Size and geometry of the source can be measured!
- Experimentally it is measured in the out − side − long pair coordinates: \( R_{x,y,z} \rightarrow R_{o,s,l} \)
- The difference between out and side radii is indicate the kind of phase transition
Azimuthally sensitive HBT radii

The transverse angle which is appear in momentum space: \((p_t, \alpha, p_z)\).

\[
R_o^2 = \langle x_o^2 \rangle - \langle x_o \rangle^2, \quad R_s^2 = \langle x_s^2 \rangle - \langle x_s \rangle^2
\]

where: \(x_o = r \cos(\varphi - \alpha)\), \(x_s = r \sin(\varphi - \alpha)\)

The average is an integrating over the source function with weight \(x_o\) or \(x_s\) respect the spatial coordinates \((r, \varphi, r_z)\)

Parametrization: Elliptical case: \(R_{o/s}^2 = R_{o/s,0}^2 + R_{o/s,2}^2 \cos(2\alpha)\)

Parametrization: Triangular case: \(R_{o/s}^2 = R_{o/s,0}^2 + R_{o/s,3}^2 \cos(3\alpha)\)
Azimuthally sensitive HBT radii

Mixing of the parameters: spatial distribution and velocity field form the azimuthally sensitive HBT radii together.

Elliptical case: in the second order reaction plane

$\epsilon_2$: asymmetry in space-time, $\chi_2$: asymmetry in velocity field

Parametrization: Elliptical case: $R_{o/s}^2 = R_{o/s,0}^2 + R_{o/s,2}^2 \cos(2\alpha)$
Azimuthally sensitive HBT radii

Mixing of the parameters: spatial distribution and velocity field form the azimuthally sensitive HBT radii together.

Triangular case: in the third order reaction plane 

$\epsilon_3$: asymmetry in space-time, $\chi_3$: asymmetry in velocity field 

Parametrization: Triangular case: 

$$ R^2_{o/s} = R^2_{o/s,0} + R^2_{o/s,3} \cos(3\alpha) $$

![Graphs showing $R_{out,3}$ and $R_{side,3}$ vs $\chi_3$ and $\epsilon_3$]
Conclusion and outlook

- Hydrodynamical approach can be used as a phenomenological tool to describe QCD matter
- The geometry of the source can be investigated
- General asymmetries can be built into a model
- Observables can be derived from the generalized model
- No mixing between 2\textsuperscript{nd} and 3\textsuperscript{rd} order asymmetries
- There is mixing between the spatial and velocity field asymmetries
- It is important to explore these mixing based on a realistic model

THANK YOU FOR YOUR ATTENTION!
Values of the parameters

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Effect of the parameter on the source

$\varepsilon_2=0.8, \varepsilon_3=0, \varepsilon_4=0$

$\varepsilon_2=0.8, \varepsilon_3=0.5, \varepsilon_4=0$

$\varepsilon_2=0.8, \varepsilon_3=0.5, \varepsilon_4=0.4$