Di-Muon production in heavy ion collisions at LHC: A signal for Quark-Gluon deconfinement

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Electromagnetic probes.

arXiv:0905.0174 by P. Sorensen
ELECTROMAGNETIC PROBES.

Nuclear collisions and the QGP expansion

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The correlation function of these currents is introduced and treated in the framework of the OPE.

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\Pi^\text{QCD}_0(Q^2) = C_0 \hat{I} + \sum_{N=1} C_{2N}(Q^2, \mu^2) \frac{Q^{2N}}{Q^{2N}} \langle \hat{O}_{2N}(\mu^2) \rangle.
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\[
\begin{align*}
\text{QCD Quark-Propagator} & = \underbrace{\text{perturbative contribution}}_{\propto \langle \bar{q} q \rangle} + \underbrace{\text{non-perturbative contribution}}_{\propto \langle g_\pi \sigma G q \rangle + \langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle + \ldots}
\end{align*}
\]
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Finite Energy QCD Sum Rules (FESR).
Time to join both sectors!!!

\[ \int_0^{s_0} ds P(s) \frac{1}{\pi} \text{Im} \Pi(s) = -\oint_{C(|s_0|)} ds P(s) \Pi^{OPE}(s). \]
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\[ (-1)^N C_{2N+2} \langle \hat{O}_{2N+2} \rangle = \int_0^{s_0} ds \ s^N \frac{1}{\pi} \text{Im} \Pi^{HAD}(s) \]
\[ + \frac{1}{2\pi i} \oint_{C(|s_0|)} ds \ s^N \Pi^{QCD}(s). \]
We work with QFT at finite temperature.

\[
S_F(T = 0) = \frac{k + m}{k^2 - m^2 + i\epsilon},
\]

\[
S_F(T) = S_F(T = 0) + 2\pi i \delta(k^2 - m^2)(k + m)n_F(|k_0|),
\]

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D_B(T = 0) = \frac{i}{p^2 - m^2 + i\epsilon},
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Hadronic parameters develop thermal behavior (Masses, coupling constants, resonance’s widths).
The parameter $s_0$ is thermal-dependent.

Qualitative parameter of deconfinement.
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$\text{Im}\Pi^{\text{HAD}}(s)$ is related with the hadronic spectral function and the latter is well approximated by the Breit-Wigner form

$$\frac{1}{\pi}\text{Im}\Pi^{\text{HAD}}(s) = \frac{1}{\pi} \frac{1}{f_{\rho}^2} \frac{M_{\rho}^3 \Gamma_{\rho}}{(s - M_{\rho}^2)^2 + M_{\rho}^2 \Gamma_{\rho}}.$$ 

Finite Sum Rules at finite temperature.

$$(-1)^{N-1} C_{2N} \langle O_{2N} \rangle = 8\pi^2 \left[ \int_{0}^{s_0} ds \ s^{N-1} \frac{1}{\pi} \text{Im}\Pi^{\text{HAD}}(s) - \frac{1}{2\pi i} \oint_{|s_0|} ds \ s^{N-1} \Pi^{\text{QCD}}(s) \right].$$
Dimuon production from in-medium $\rho$ decays.

The solution from FESR for all hadronic parameters as a function of $T$ are the following

\begin{align*}
\Gamma_\rho(T) &= \Gamma_\rho(0)[1-(T/T_c)^3]^{-1}, \\
M_\rho(T) &= M_\rho(0)[1-(T/T_M^*)^{10}], \\
f_\rho(T) &= f_\rho(0)[1-0.3901(T/T_c)^{10.75} \\
&+ 0.04155(T/T_c)^{1.27}].
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Temperature behaviour of $f_\rho$

Temperature behaviour of $\Gamma_\rho$
With the solution from the FESR, we proceed to compute the dimuon thermal rate in the hadronic phase originating from $\rho$ decays. (We consider processes where pions annihilate into $\rho$ which in turn decay into dimuons by means vector dominance.)

\[
\frac{dN}{d^4x d^4K} = \frac{\alpha^2}{48\pi^4} \left( 1 + \frac{2m^2}{M^2} \right) \left( 1 - \frac{4m^2}{M^2} \right) \times \sqrt{1 - \frac{4m^2}{M^2}} e^{-K_0/T} \mathcal{R}(K,T) \text{Im} \Pi^\text{res}_0(M^2),
\]
Dimuon production from in-medium $\rho$ decays.

Non-model dependence result but directly from the perturbative and non-perturbative QCD information.

Invariant dimuon mass distribution compared to NA60 data.

(Linear scale) Invariant dimuon distribution around $\rho$-meson peak compared to NA60 data.
Thank you!!!

For more information and details:

QCD Sum Rules was developed more than 30 years ago by Shifman, Vainshtein and Zakharov (SVZ).

The light-quark vector current correlator, which at \( T = 0 \) can be written as

\[
\Pi_{\mu\nu}(q^2) = i \int d^4 x e^{i q \cdot x} \langle 0 \mid T \left[ \mathcal{V}_\mu(x) \mathcal{V}_\nu^\dagger(0) \right] \mid 0 \rangle
\]

\[
= (-g_{\mu\nu} + q_\mu q_\nu) \Pi_1(q^2),
\]

where \( \mathcal{V}_\mu(x) = (1/2) \left[ : \bar{u}(x) \gamma_\mu u(x) - \bar{d}(x) \gamma_\mu d(x) : \right] \) is the conserved vector current and \( q_\mu \) is the four-momentum transfer.

In the thermal perturbative QCD sector, only one-loop contributions can be taken into account, since the problem of the appearance of two scales, i.e. the short-distance QCD scale and the critical temperature \( T_c \), remains unsolved.
\( \Gamma_\rho(0) = 0.145 \text{ GeV}, M_\rho(0) = 0.776 \text{ GeV}, T_c = 0.197 \text{ GeV} \) and \( f_\rho(0) = 5 \)

In order to extend this analysis to finite chemical potential we first incorporate the \( \mu \) dependence into the pQCD sector, which involves a quark loop. This modifies the corresponding Fermi-Dirac distribution, splitting it into particle-antiparticle contributions. And we incorporate the \( \mu \) dependence of the critical temperature \( T_c \). For this, we use a Schwinger-Dyson approach, a parametrization for the crossover transition line between chiral-symmetry-restored and -broken phases.

\[
T_c(\mu) = T_c(\mu = 0) - 0.218 \mu - 0.139 \mu
\]
The rate is given by

\[
\frac{dN}{d^4x d^4K} = \frac{\alpha^2}{48\pi^4} \left(1 + \frac{2m^2}{M^2}\right) \left(1 - \frac{4m^2_{\pi}}{M^2}\right) \times \sqrt{1 - \frac{4m^2}{M^2}} e^{-K_0/T} \mathcal{R}(K,T) \text{Im} \Pi_0^{\text{res}}(M^2),
\]

where \( N \) is the number of muon pairs per unit of infinitesimal space-time and energy-momentum volume, with \( x^\mu \) the space-time coordinate and \( K^\mu \) the four-momentum of the muon pairs, \( \alpha \) is the electromagnetic coupling, \( m \) is the muon mass, \( m_{\pi} \) is the pion mass and \( M \) is the dimuon invariant mass. And

\[
\mathcal{R} = \frac{T/K}{1 - e^{-K_0/T}} \times \ln \left[ \left(\frac{e^{-E_{\max}/T}}{e^{-E_{\min}/T} - 1}\right) \left(\frac{e^{E_{\min}/T} - e^{-K_0/T}}{e^{E_{\max}/T} - e^{-K_0/T}}\right) \right],
\]
with

\[
E_{\text{max}} = \frac{1}{2} \left[ K_0 + K \sqrt{1 - 4m^2_\pi / M^2} \right]
\]

\[
E_{\text{min}} = \frac{1}{2} \left[ K_0 - K \sqrt{1 - 4m^2_\pi / M^2} \right].
\]

In order to integrate the dimoun thermal rate, we use

\[
dx^4 = \frac{1}{2} dM^2 d^2 K_\perp dy
\]

\[
dx^4 = \tau d\tau d\eta d^2 x_\perp,
\]

where \( y \) and \( \eta \) are the momentum-space and coordinate-space rapidities, respectively and \( \tau = \sqrt{t^2 - z^2} \). To relate the temperature change to the time evolution of the system, we neglect a possible small transverse expansion, assume it entirely longitudinal, and use the cooling law

\[
T = T_0 \left( \frac{\tau}{\tau} \right)^{v_s^2},
\]
where $v_s^2 = 1/3$ is the square of the sound velocity for an ideal hadron gas. The evolution is taken down to a freeze-out temperature $T_f$. Also, we consider perfect correlation between $\eta$ and $y$ ($\eta = y$). The invariant mass distribution becomes

$$
\frac{dN}{dM dy} = \Delta y M \int_{\tau_0}^{\tau_f} \tau d\tau \int d^2 K_{\perp} \int d^2 x_{\perp} \frac{dN}{d^4 x d^4 K}.
$$