Description of baryons in composite superconformal string model.

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Our aim and instrument

- Description of meson and baryon spectrum.
- Construction of interaction amplitude at low and intermediate energies (0.1 - 7 GeV).
- String has superconformal (gauge) symmetry (Super Virasoro algebra $L_n, G_r$).
- New type of string model.
  - Slope of Regge trajectories $\alpha'$ is about usual hadron scale $1\text{GeV}^{-2}$.
  - Intercept $\alpha_0$ of leading meson trajectory is equal to $\frac{1}{2}$.
  - New topology. Additional two-dimensional surfaces.
  - Supersymmetry occurs on two-dimensional world surface only. Target space does not have supersymmetry.
Description of fermions in this model

- Basic two-dimensional surface. $\partial X_\mu, H_\mu, I, \theta$.
- Additional two-dimensional surfaces. $Y^{(i)}_\mu, f^{(i)}_\mu, J^{(i)}, \Phi^{(i)}$.
- Quark spinors and isospinors $\lambda_i$ are represented by eigenvectors of zeroth component of field $J^{(i)}$. $J^{(i)}_0 \lambda_i = \xi_i \lambda_i$.

Figure: Composite string for baryon.
Direct amplitude

$\partial X^\mu \ H_\mu$

I $\theta$

Figure: N interaction.

A.N. Semenova  String description of baryons
Vertex operator formalism

- Two-dimensional superconformal symmetry.
  Quantum super Virasoro algebra:
  \[
  [L_n, L_m] = (n - m)L_{n+m} + \delta_{n,-m}c_1 n(n^2 - 1),
  \{ G_r, G_s \} = 2L_{r+s} + c_2 (r^2 - 1/4) \delta_{r,-s},
  [L_n, G_r] = (n/2 - r) G_{n+r}.
  \]

- We formulate vertex operator $\hat{\mathcal{V}}$ of ground state emission which satisfies superconformal symmetry:
  \[
  \hat{\mathcal{V}}(z_i) = z_i^{-L_0} \left[ G_r, \hat{\mathcal{W}} \right] z_i^{L_0}, \quad \hat{\mathcal{W}} \sim: e^{-ikX} :.
  \]

- We use formalism of vertex operator with conformal spin $J = 1$.
  \[
  [L_n, \hat{\mathcal{V}}_{J=1}] = \frac{d}{d\tau} z^n \hat{\mathcal{V}}_{J=1}, \quad z = e^{i\tau}
  \]

- Additional supercurrent conditions to eliminate all negative norms from physical state spectrum.
  \[
  [k_i Y_n^{(i)}, \hat{\mathcal{W}}_{i,i+1}] = [\hat{\mathcal{W}}_{i,i+1}, k_{i+1} Y_n^{(i+1)}] = 0.
  \]
Spin and isospin structure

Figure: Schematic $\pi$ N interaction diagram.

- Interaction amplitude for $\pi$-meson and nucleon:
  \[ A_{\pi N} = \int dz \langle 0 | \hat{V}_N \hat{V}_\pi \hat{V}_\pi \hat{V}_N | 0 \rangle. \]

- Spin structure
  - $\pi$-meson spin-parity $0^-$: $(\lambda_3 \gamma_5 \lambda_4)$
  - Nucleon spin-parity $\frac{1}{2}^+$:
    \[ V_N^{(NS)} : (\lambda_1 \gamma_5 \lambda_3) \lambda_f \gamma_5, \quad V_N^{(BH)} : (\lambda_1 \lambda_3) \lambda_f, \]

- Isospin structure
  - $\pi$-meson isospin $T = 1$: $V_\pi : \lambda_3 \tau^{(i)} \lambda_4$.
  - Nucleon isospin $T = \frac{1}{2}$: $V_N : (\lambda_1 \lambda_3) \lambda_f$
Interaction amplitude of $\pi$ meson and nucleon

For pole contributions in $t$-channel near pole values $t = m_i^2$ (mass condition $L_0 = 1$) we can consider direct interaction amplitude of $\pi N$:

$$A_{\pi N} \sim \Pi_+(t) \frac{\Gamma(\frac{1}{2} - \alpha_{N^+}(t))\Gamma(1 - \alpha^\rho(s))}{\Gamma(\frac{3}{2} - \alpha_{N^+}(t) - \alpha^\rho(s))} + \Pi_+(t) \frac{\Gamma(\frac{3}{2} - \alpha_{\Delta^-}(t))\Gamma(1 - \alpha^\rho(s))}{\Gamma(\frac{3}{2} - \alpha_{\Delta^-}(t) - \alpha^\rho(s))} +$$

$$+ \Pi_-(t) \frac{\Gamma(\frac{3}{2} - \alpha_{\Delta^+}(t))\Gamma(1 - \alpha^\rho(s))}{\Gamma(\frac{3}{2} - \alpha_{\Delta^+}(t) - \alpha^\rho(s))} + \Pi_-(t) \frac{\Gamma(\frac{3}{2} - \alpha_{N^-}(t))\Gamma(1 - \alpha^\rho(s))}{\Gamma(\frac{3}{2} - \alpha_{N^-}(t) - \alpha^\rho(s))}.$$

Where

$$\alpha_{N^+}(t) = -\frac{1}{4} + \frac{t}{2}, \quad \alpha_{N^-}(t) = -\frac{1}{4} + \frac{t}{2},$$

$$\alpha_{\Delta^+}(t) = \frac{1}{4} + \frac{t}{2}, \quad \alpha_{\Delta^-}(t) = -\frac{3}{4} + \frac{t}{2}.$$
N Regge trajectories

Green dots: $m\left(\frac{1}{2}^+\right) = 940 \text{ MeV}$, $m\left(\frac{5}{2}^+\right) = 1680 \text{ MeV}$, $m\left(\frac{9}{2}^+\right) = 2250 \text{ MeV}$.

Blue dots: $m\left(\frac{5}{2}^-\right) = 1675 \text{ MeV}$, $m\left(\frac{9}{2}^-\right) = 2220 \text{ MeV}$. 
Regge trajectories

\[ \alpha_{\Delta^+} = \frac{1}{4} + \frac{t}{2} \]

\[ \alpha_{\Delta^-} = -\frac{3}{4} + \frac{t}{2} \]

Red dots: \( m(\frac{3}{2}^+) = 1232 \text{ MeV}, \ m(\frac{7}{2}^+) = 1950 \text{ MeV}, \ m(\frac{11}{2}^+) = 2420 \text{ MeV}. \)

Violet dot: \( m(\frac{3}{2}^-) = 1700 \text{ MeV}. \)
Conclusions

- The model gives nondegenerate in parity fermion Regge trajectories.
- It is a new type of string model.
- The leading meson trajectory has the intercept $1/2$.
- Supersymmetry conditions are satisfied on the two-dimensional surface only.
- Physical spectrum of states is free from ghosts.
Thank you
Projectors on parity

- λ contains numerical spinor \( u \), satisfying Dirac equation:
  \[
  \hat{P}_f u^{(+)} = m u^{(+)}, \quad \hat{P}_f u^{(-)} = -m u^{(-)}.
  \]
  \[
  \sum \bar{u}_\alpha^{(i)} u_\beta^{(i)} + \sum \bar{u}_\alpha^{(i)} u_\beta^{(i)} = \left( \frac{m + \hat{q}}{2m} \right)_{\alpha\beta} + \left( \frac{m - \hat{q}}{2m} \right)_{\alpha\beta} = 1.
  \]
- Initial state chooses \( P_f = +1 \) or \( P_f = -1 \).
- For each pole arises projector:
  \[
  \Pi_+ = \frac{m + \hat{q}}{2m}, \quad \Pi_- = \frac{m - \hat{q}}{2m}.
  \]
- For arbitrary \( q \) we suggest analytic continuation:
  \[
  \frac{m \pm \hat{q}}{2m} \to \frac{1}{2} \left( 1 \pm \frac{\hat{q}}{\sqrt{2(R - L_0)}} \right) \to \left( \frac{1}{2} \pm \frac{\hat{q}}{\sqrt{\pi}} \int_0^\infty \! dy \ e^{-2(R-1)y^2} \right).
  \]
- We used: \( L_0 = -\frac{q^2}{2} + R \), and mass condition \( L_0 = 1 \).