The decays of a neutral particle with zero spin and arbitrary $CP$ parity into $ZZ$ or $W^- W^+$

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Introduction

- In 2012 the ATLAS and CMS collaborations observed a boson $h$ with the mass around 126 GeV. We call this particle the Higgs boson. However, clarification of properties of the observed boson $h$ requires more data.

\[ q_h = 0 \]

\[ S_h = 0 \text{ or } S_h = 2 \text{ (very unlikely)} \]

\[ CP_h =? \]

- In the SM for the Higgs boson

\[ q = 0, S = 0, C = P = 1, \]

but some supersymmetric extensions of the SM assume existence of Higgs bosons with negative or indefinite $CP$ parity.
Plan of the investigation

In order to clarify the \( CP \) properties of \( h \) the following way has been chosen.

- We consider the decay \( X \rightarrow Z_1^*Z_2^* \rightarrow f_1\bar{f}_1f_2\bar{f}_2 \), where \( X \) is a neutral particle with zero spin and arbitrary \( CP \) parity, \( f_1 \neq f_2 \).
Plan of the investigation

\[ A_{X \rightarrow Z_1^* Z_2^*} \sim a(e_1^* \cdot e_2^*) + \frac{b}{m_X^2} (e_1^* \cdot p_2)(e_2^* \cdot p_1) + i \frac{c}{m_X^2} \varepsilon_{\mu
u\rho\sigma} (p_1^{\mu} + p_2^{\mu})(p_1^{\nu} - p_2^{\nu}) e_1^{*\rho} e_2^{*\sigma} \]

\( e_1 \) and \( e_2 \) are the polarization 4-vectors of \( Z_1^* \) and \( Z_2^* \) respectively.
\( a, b, c \) are complex-valued functions of the masses of \( Z_1^* \) and \( Z_2^* \). These functions characterize the \( CP \) properties of the boson \( X \). At tree level

<table>
<thead>
<tr>
<th>( CP_X )</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>any</td>
<td>any</td>
<td>0</td>
</tr>
<tr>
<td>1 (SM)</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>( \neq 0 )</td>
</tr>
<tr>
<td>indefinite</td>
<td>( \neq 0 )</td>
<td>any</td>
<td>( \neq 0 )</td>
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<td></td>
<td>any</td>
<td>( \neq 0 )</td>
<td>( \neq 0 )</td>
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</tbody>
</table>

- We derive the full distribution of the decay \( X \rightarrow Z_1^* Z_2^* \rightarrow f_1 \bar{f}_1 f_2 \bar{f}_2 \).
- Experimentalists measure an experimental full distribution of this decay for \( X = h \).
- Comparing the theoretical and experimental distributions, one can get constraints on the values of \( a, b, c \) at various masses of \( Z_1^* \) and \( Z_2^* \).
Definitions of $\theta_1$, $\theta_2$, $\varphi$

$\theta_1$ is the angle between the momentum of $Z_1^*$ in a rest frame of $X$ and the momentum of $f_1$ in a rest frame of $Z_1^*$,

$\theta_2$ is the angle between the momentum of $Z_2^*$ in a rest frame of $X$ and the momentum of $f_2$ in a rest frame of $Z_2^*$,

$\varphi$ is the azimuthal angle between the planes of the decays $Z_1^* \rightarrow f_1 \bar{f}_1$ and $Z_2^* \rightarrow f_2 \bar{f}_2$. 
Definitions of $A_0$, $A_∥$, $A_⊥$

Moreover, it is convenient to write down the fully differential width by means of the following amplitudes:

$$A_0 \equiv -\left( a \frac{m_X^2 - a_1 - a_2}{2\sqrt{a_1a_2}} + b \frac{\lambda(m_X^2, a_1, a_2)}{4m_X^2 \sqrt{a_1a_2}} \right),$$

$$A_∥ \equiv \sqrt{2}a,$$

$$A_⊥ \equiv \sqrt{2}c \frac{\lambda^{\frac{1}{2}}(m_X^2, a_1, a_2)}{m_X^2}.$$

$a_j$ is the mass squared of $Z_j^*$, i.e. the invariant mass of the pair $f_j\bar{f}_j$, $\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.$
The differential width with respect to $a_1$, $a_2$, $\theta_1$, $\theta_2$, $\varphi$

Using approximations $m_{f_1} = m_{f_2} = 0$, we have derived that

$$d^5\Gamma \over da_1 da_2 d\theta_1 d\theta_2 d\varphi = |A_0|^2 f_1 + (|A_\parallel|^2 + |A_\perp|^2) f_2 + (|A_\parallel|^2 - |A_\perp|^2) f_3$$

$$+ \text{Re}(A_0^* A_\parallel) f_4 + \text{Re}(A_0^* A_\perp) f_5 + \text{Re}(A_\parallel^* A_\perp) f_6$$

$$+ \text{Im}(A_0^* A_\parallel) f_7 + \text{Im}(A_0^* A_\perp) f_8 + \text{Im}(A_\parallel^* A_\perp) f_9.$$ 

$f_1$, $f_2$, ..., $f_9$ depend on $a_1$, $a_2$, $\theta_1$, $\theta_2$, $\varphi$, but they are independent of $a$, $b$ and $c$.

The dependence of the fully differential width on the couplings $a$, $b$ and $c$ is concentrated in nine quadratic combinations of the amplitudes $A_0$, $A_\parallel$, $A_\perp$.

How many decays should be measured for obtaining a precise enough experimental full distribution of the decay?

$$d^n\Gamma \leftrightarrow 10^{n+1} \text{ decays}$$

$$d^5\Gamma \leftrightarrow 10^6 \text{ decays}$$

How many decays have been observed?

$$h \rightarrow Z_1^* Z_2^* \rightarrow e^- e^+ \mu^- \mu^+$$

26 decays (ATLAS and CMS together after about 1.5 years of measurements)
Distributions of four and less variables should be considered.

We will probably have a precise enough experimental full distribution in 60000 years (roughly).

That is why we should try to get constraints on $a$, $b$, $c$ by means of measuring distributions of as little a number of variables as possible.
$a_1 a_2$-differential width

Figure: $\frac{d^2\Gamma}{da_1 da_2}$ of the decay $X \rightarrow Z_1^* Z_2^* \rightarrow l_1^- l_1^+ l_2^- l_2^+$ as a function of $\sqrt{a_1}$, $\sqrt{a_2}$, if $X$ is the SM Higgs boson and $m_X = 125.7$ GeV. $l_1, l_2 = e, \mu, \tau, l_1 \neq l_2$. 

\[
\frac{d^2 \Gamma (a_1, a_2)}{da_1 da_2} \times 10^{-14} \frac{1}{\text{GeV}^3}; \quad |a_1| = 1, b = 0, c = 0
\]
Integrating $\frac{d^2\Gamma}{da_1 da_2}$ approximately, we derive that

$$d\Gamma \approx \frac{\sqrt{2} G_F^3 m_Z^9}{288 \pi^4 m_X^3 \Gamma_Z} (a_f^2 + v_f^2) (a_2^2 + v_2^2) \frac{\lambda^2 (m_X^2, m_Z^2, a_2) a_2}{(a_2 - m_Z^2)^2 + (m_Z \Gamma_Z)^2} \sum_{\lambda=0,||,\perp} |A'_\lambda|^2$$

$$\forall a_2 \mid 2m_f < \sqrt{a_2} \leq m_X - \sqrt{m_Z^2 + 3m_Z \Gamma_Z}.$$

$a_f$ and $v_f$ are constants depending on a fermion $f$, $A'_\lambda \equiv A_\lambda \mid_{a_1=m_Z^2}$. In several articles the formula for $\frac{d\Gamma}{da_2}$ has been used in the narrow-$Z$-width approximation when

$$\sqrt{a_2} \leq m_X - m_Z,$$

and their approach is inaccurate.

$$m_h - \sqrt{m_Z^2 + 3m_Z \Gamma_Z} \approx 30.8 \text{ GeV}$$
$$m_h - m_Z \approx 34.5 \text{ GeV}$$
We call the ratios of the nine quadratic combinations of the fully differential width to $\sum_{\lambda=0,\|,\perp} |A_{\lambda}|^2$ ‘the helicity coefficients’. Integrating $\frac{d^5 \Gamma}{da_1 da_2 d\theta_1 d\theta_2 d\varphi}$, we can relate all the helicity coefficients to observables. For example,

\[
O_{1}^{(1,2)}(a_2) \equiv \left( \frac{d\Gamma}{da_2} \right)^{-1} \left( \int_0^{\frac{\pi}{2}} d\theta_1,2 \frac{d^2 \Gamma}{da_2 d\theta_1,2} - \int_{\frac{\pi}{2}}^{\pi} d\theta_1,2 \frac{d^2 \Gamma}{da_2 d\theta_1,2} \right) \sim \frac{\text{Re}(A'_\|A'_\perp)}{\sum_{\lambda} |A'_{\lambda}|^2} = \text{Re}(F'_\|F'_\perp).
\]

\[
F'_\lambda \equiv \frac{A'_{\lambda}}{\sqrt{\sum_{\lambda} |A_{\lambda}|^2}}, \quad \lambda = 0, \|, \perp.
\]
Relations between the helicity coefficients and observables

\[
\begin{align*}
O_1^{(1,2)}(a_2) & \sim \Re(F_\parallel' F_\perp') \\
O_2^{(1,2)}(a_2) & \sim |F_0'|^2 \\
O_3(a_2) & \sim |F'|^2 + |F_\perp'|^2 \\
O_4(a_2) & \sim |F'|^2 - |F_\perp'|^2 \\
O_5(a_2) & \sim \Im(F_\parallel' F_\perp') \\
O_6(a_2) & \sim \Re(F_0' F_\parallel') \\
O_7^{(1,2)}(a_2) & \sim \Im(F_0' F_\parallel') \\
O_8^{(1,2)}(a_2) & \sim \Re(F_0' F_\perp') \\
O_9(a_2) & \sim \Im(F_0' F_\perp')
\end{align*}
\]

\[
\rightarrow \text{Constraints on } a, b, c
\]
Conclusions

• In order to clarify the $CP$ properties of the Higgs boson we have considered the fully mass and angular differential width of the decay $X \rightarrow Z_1^* Z_2^* \rightarrow f_1 \bar{f}_1 f_2 \bar{f}_2$, where $X$ is a neutral particle with zero spin and arbitrary $CP$ parity, $f_1 \neq f_2$.

• Limits of applicability of approximations used when deriving various differential widths are established.

• All the helicity coefficients are related to observables. We have also plotted the observables and determined what constraints on $a$, $b$, $c$ can be put by them.

• We should wait for experimentalists measuring the observables $O_1^{(1,2)}$, $O_2$, ..., $O_9$ and then get constraints on $a$, $b$, $c$ using the shown relations between $F'_0$, $F'_\parallel$, $F'_\perp$ and $O_1^{(1,2)}$, ..., $O_9$.

• An analogous analysis has been carried out for the decay $X \rightarrow W^- W^+ \rightarrow f_1 - \bar{f}_2 - \bar{f}_1 + f_2$.

The presentation is based on the paper Zagoskin and Korchin, arXiv:1504.07187.