1. Exceptional Lie groups and normed division algebras

(a) Normed division algebras and split composition algebras

The octonions can be defined by $O = \mathbb{R} \langle 1, e_1, \ldots, e_7 \rangle$, with $e_1$ imaginary, $e_2 \cdot e_1 = -e_1 \cdot e_2 = e_3$, and multiplication rule described by the Fano plane:

$$e_1 e_2 = e_3, \quad e_2 e_3 = e_4, \quad e_3 e_4 = e_5, \quad e_4 e_5 = e_6, \quad e_5 e_6 = e_7, \quad e_6 e_7 = e_1$$

The split octonions $O_D$ can be obtained e.g. by substituting $e_i \rightarrow x_i$, $i = 1, \ldots, 7$, so that $x_7 = e_1$ instead of $x_1 = -1$.

Subalgebras corresponding to the quaternions $A$ and the complex numbers $C$ are e.g. $B = \mathbb{R} \langle 1, e_1, e_2, e_3 \rangle$.

(b) Vinberg's formula

Vinberg's formula (Wonchok, Vinberg, Lie Groups and Lie Algebras III. Springer, Berlin, 2004) allows to determine all the Lie algebras $\mathfrak{g}$ of the above exceptional Lie algebras. I have also developed an algebraic method to determine the global structure of the full group, which for the compact form is a generalization of the Euler angles for $\mathfrak{g}(2)$, while for the non compact forms it is based on the (real) decomposition.

For the last several years I have been pursuing a project of mapping the geometry of the exceptional Lie groups $\mathfrak{g}(2, \mathbb{R})$, and how they act as symmetries of different physical models. The method used to construct the corresponding Lie algebras utilizes the Tits formula, which exploits the Jordan algebra structure: for a group of matrices $G$ is the group of all matrices conjugating the given matrix to diagonal form.

I have written a Mathematica program to generate the structure and the finite-dimensional irreducible representations and the adjoint of each of the exceptional Lie algebras.

2. Global parametrizations of Lie groups: The Lie algebra parametrization for the noncompact form and the Euler angles for the compact form

3. Applications to supergravity

Exceptional Lie groups are relevant for extended supergravity theories, where they enter as the electric-magnetic duality, or the stabilizer/isotropy groups of scalar manifolds and of certain black hole solutions. The Euler angles determine the structure of the exceptional Lie groups.

The double split Barton-Sudbery magic square $L^3(\mathbb{AS}, \mathbb{BS})$ allows to recover also $A^\sim(8)$ and $E^\sim(14)$.

(d) Iwasawa parameterization of $E^{\sim}(7)$

It is relevant for $A^\sim(N)$ in $D = 8 \times 8$-space-time dimensions [Greiner, Jaffa, Nucl. Phys. B159, 191 (1979)].

4. Application to quantum information theory and the embedding of $\mathfrak{sl}(2)^{7\times}$ in $\mathfrak{e}^\sim(7)\sim$

5. Study of $\mathfrak{sl}(2)$ subgroups

6. Conclusions and outlook