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Microstates in Black Holes

and

the Topology of Space-Time

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At first sight, the gravitational force seems to be just like any other force among elementary particles.

It has the same $1/r^2$ dependence, and it is based on a local gauge principle,

just like electromagnetism, the weak force and the strong force.

Subtle differences: the carrier (graviton) has spin $= 2$, while photons have spin $= 1$; and

Gravity acts on *mass*, which is like *energy*,
so for high energy particles, gravity becomes *very strong*.

Standard procedures (quantum field theory) therefore fail.

Curvature of space-time is too much to handle.
Last 40 years' strategy:

> "guess the correct answer, close your eyes, and jump"

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Are there such spots?

Yes!

Whenever a **black hole** forms,
and we want to apply QM !

Schwarzschild, Reissner-Nordström, Kerr, Kerr-Newman BH:

What does the text book say?

Hawking 1975: BH emits particles! However:

BH consists of interior part and exterior part.
Particles entering the interior part cannot come out.
Their quantum information is lost. Therefore, particles emerging from BH are in a **mixed state**.

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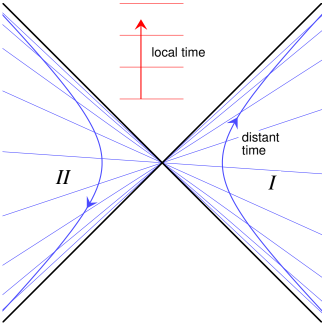

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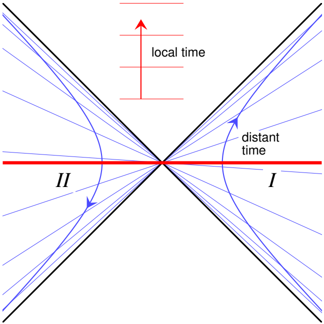

Hartle-Hawking vacuum:

$$|\,HH\,\rangle \;=\; C\sum_{E,n} e^{-\frac{1}{2}\beta E}\,|E,n\rangle_I\,|E,n\rangle_{II}$$

I = outside
II = inside [?]

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so, people start guessing again

. . . What does superstring theory say?

They find something that looks like a black hole,
and it is described in terms of pure quantum states !
"microstates". But it is not understood . . .

How are these microstates related to vacuum fluctuations?

The string theory "text book" is very unclear about these issues.
It claims that Hawking radiation must be in a pure state. But then:

Difficulties regarding entanglement and no-cloning:

Almheiri, Marolf, Polchinski, Sully (AMPS): *If we start in a pure state, and consequently Hawking radiation is is a pure state, then that pure state must be entangled also with earlier radiation. Therefore it can't be in the state originally used by Hawking. This produces a curtain of infinitely energetic particles along the future event horizon: a firewall*

This cannot be right.
As we discovered, you can do better.

But yes, there will be new physics.

We begin with getting the maximum out of *standard Einstein General Relativity* and *standard Quantum Mechanics*

Then, one has to discover 3 important things:

(i) Particles going into a black hole, interact *gravitationally* with particles going out. If you want to describe these as pure quantum states, you cannot ignore that (only if they are in mixed states, you may) because this grav. force is **strong**

(ii) This force generates an algebra that is *linear* in the coordinates & momenta of the in- and out-particles, and therefore, you can superimpose solutions!

$\longrightarrow$ Make an expansion in spherical haronics.

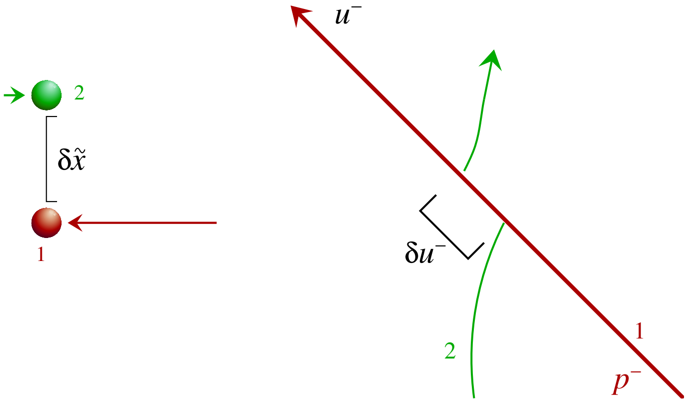
(iii) We had always been wrong in thinking that Hawking particles going into the BH were lost. They reappear at the other side!

If we ignore any of these 3 points, we fail to understand what happens

The gravitational backreaction:

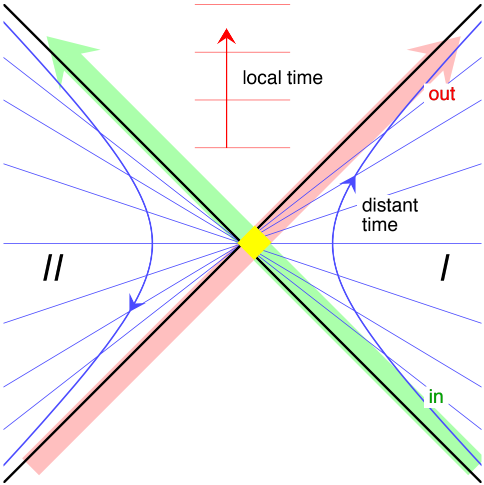
calculate the grav. field due to a fast moving particle,

simply Lorentz boost the field of a particle at rest:



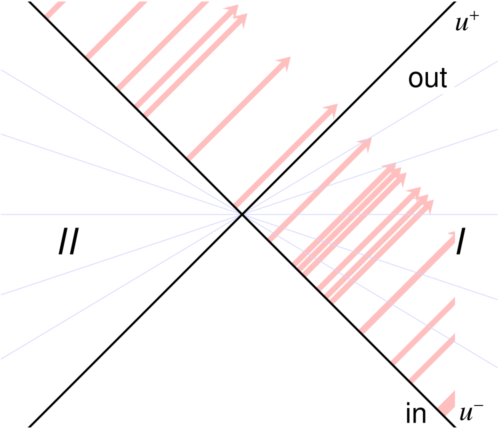
$$\delta u^-(\tilde{x}) = -4G\, p^-(\tilde{x}\,')\, \log|\tilde{x} - \tilde{x}\,'|\ .$$

Classically: out-going particles are independent of in-going ones.



But not in black holes. This used to be the BH information paradox.

An extra particle $\delta(\mathrm{in})$ with momentum δp^- going in interacts gravitationally with the out-going particles, causing a shift δu^-:



$$\delta u^-(\Omega) \approx -4G\,\delta p^-(\Omega')\,\log|\Omega - \Omega'|\ , \qquad \Omega \equiv (\theta,\,\varphi)\ .$$

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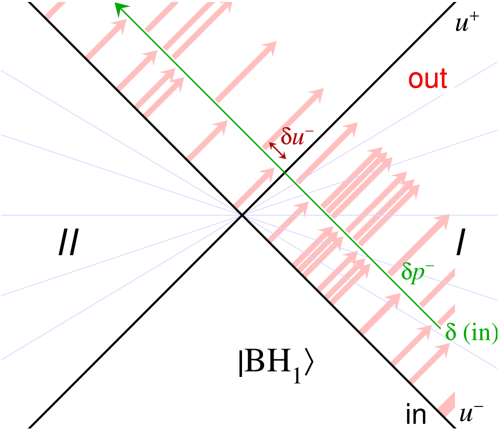

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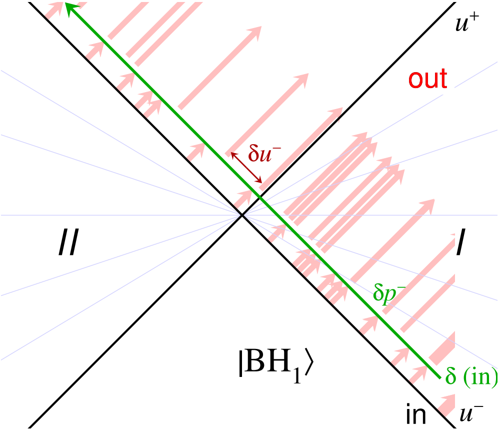

$\delta u^-(\Omega) \approx -4G\,\delta p^-(\Omega')\,\log|\Omega - \Omega'|\ .$

If you wait a few more microseconds, the in-going particles distorts the out-going particle state so much that they may come out of the past event horizon heavily mutilated.

There is a 'curtain', or, a firewall, along the past horizon.

Similarly, particles emerging in the later future may cause a 'firewal' in the near future.

If we replace mixed quantum states with pure quantum states, should such firewalls not necessarily appear?

. . . arguments concerning *entangled* Hawking particles . . .

This would imply that the extra regions in the Penrose diagram cannot be reached at all.

Our new theory explains what happens. Firewalls can be transformed away.

All in-going particles add (linearly) to this grav. drag effect. and all out-going particles are dragged.

The sum of all momenta going in, drag the average positions of all out-going things.

This leads to a *linear* algebra relating the momenta of the in-going particles to the positions of the out-going ones.

These obey simple commutation relations,

$$[\,\tfrac{1}{N}\sum_i u_i\,,\ \sum_j p_j\,] = i$$

Now, integrate all particles over the horizon:

An algebra for the total momentum densities and the average position operators, as distributions on the horizon:

$$\begin{aligned} u^-_{\rm out}(\Omega) &= \int \mathrm{d}^2\Omega'\, f(\Omega,\,\Omega')\, p^-_{\rm in}(\Omega')\ , \\ (\Delta_\Omega - 1)\, f(\Omega,\,\Omega') &= -8\pi G\, \delta^2(\Omega,\,\Omega') \end{aligned}$$

$$\begin{aligned} [u^-_{\rm out}(\Omega),\ p^+_{\rm out}(\Omega')] &= [u^+_{\rm in}(\Omega),\ p^-_{\rm in}(\Omega')] \ = \ i\,\delta^2(\Omega,\,\Omega')\ . \\ [u^+_{\rm in}(\Omega), u^-_{\rm out}(\Omega')] &= i f(\Omega,\,\Omega') \\ (\Delta_\Omega - 1) u^-_{\rm out}(\Omega) &= -\,8\pi G p^-_{\rm in}(\Omega)\ ; \\ (\Delta_\Omega - 1) u^+_{\rm in}(\Omega) &= +\,8\pi G p^+_{\rm out}(\Omega)\ ; \end{aligned}$$

This algebra is extremely simple, but also tricky.
How to interpret the sign switch $\mathrm{in} \leftrightarrow \mathrm{out}$?
(it *is* correct, in our notation)

All states, both in the initial and the final black hole, are a representation of this algebra.

The relation between in- and out- is now not much more than a Fourier transformation: $u^{\pm} \leftrightarrow p^{\mp}$, with $p = -i\frac{\partial}{\partial u}$

Is that all ? **NO !**

There is a complication.

Let's calculate the representation:

Do the partial wave expansion

Partial waves on the spherical black hole:

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$$\begin{pmatrix} u \\ p \end{pmatrix}(\theta, \varphi) = \sum_{\ell,m} \begin{pmatrix} u_{\ell,m} \\ p_{\ell,m} \end{pmatrix} Y_{\ell,m}(\theta, \varphi)$$

$$[u^{\pm}_{\ell,m}\,,\ p^{\mp}_{\ell',m'}] = i\delta_{\ell\ell'}\,\delta_{mm'}$$

$$(1 - \Delta_\Omega) f(\Omega) = 8\pi G\, \delta^2(\Omega) \rightarrow (\ell^2 + \ell + 1) f_{\ell,m} = 8\pi G\ .$$

$$\boxed{u^{\pm}_{\ell,m} = \mp \frac{8\pi G / R^2}{\ell^2 + \ell + 1}\, p^{\pm}_{\ell,m}}$$

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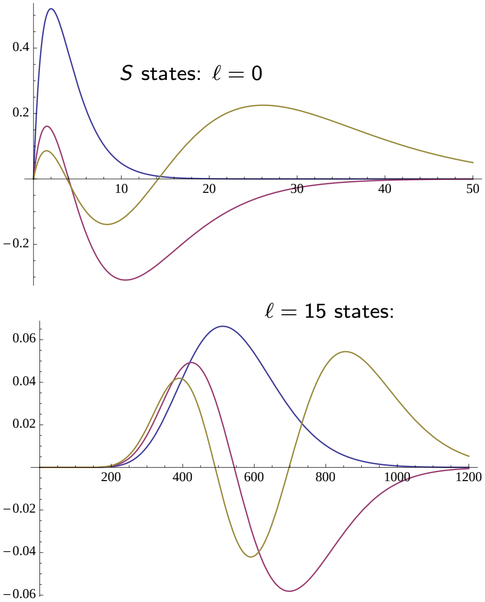
The out-going wave is the Fourier transform of the in-going wave.

Indeed, if $u_{\rm in}$ approaches horizon at $u=0$ as: $u_{\rm in}\to\lambda u_{\rm in}$, then $u_{\rm out}$ will be driven out: $u_{\rm out}\to\lambda^{-1}u_{\rm out}$

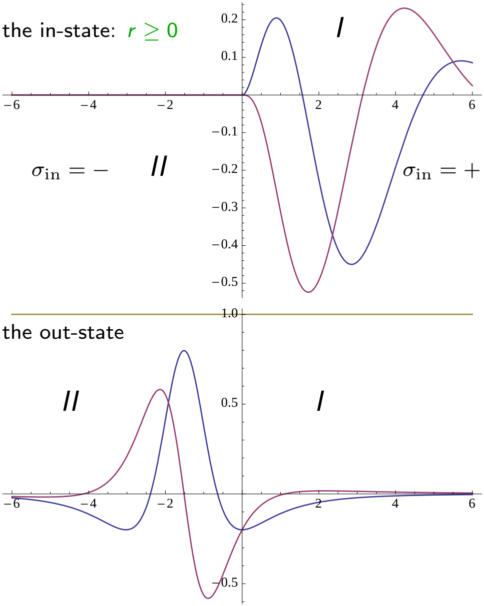
As in the hydrogen atom: the effects caused by the Schrödinger equ. become transparent if you look at modes with given ℓ, m.

Radius $r \geq 0$.

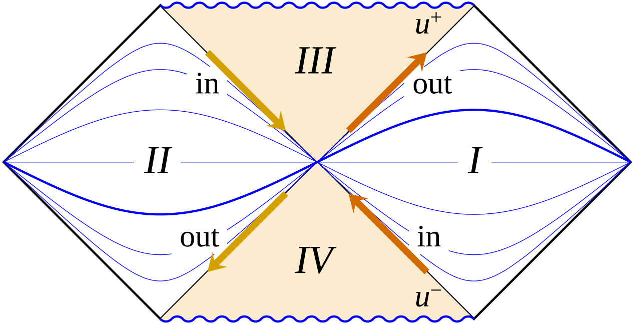


Partial wave expansion in black hole: at given ℓ, m, the out-state is the Fourier transform of the in-state.

However, Radius $r \geq 0$ and $r < 0$ both occur!



The Penrose diagram



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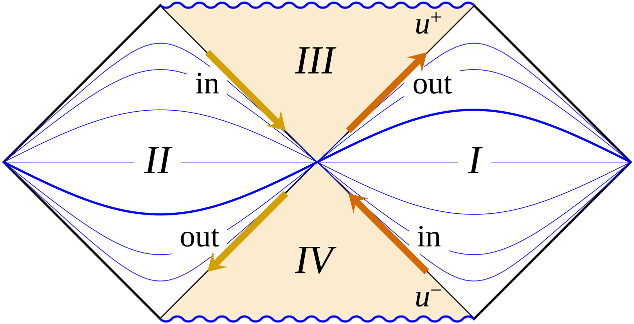

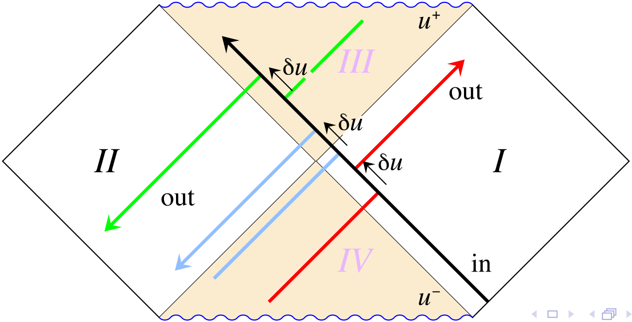

In terms of the coordinates u^+ and u^-, these wave functions were each other's Fourier transform.

But, for distant observers, $u^\pm$ and $p^\pm$ depend exponentially on time $\tau = t/4GM$: $\quad u^\pm,\ p^\pm \ \to\ e^{\mp\tau}$

Introduce tortoise coordinates (close to horizon)

$$u_{\rm in}^+ = \sigma_{\rm in}\, e^{\varrho_{\rm in}}\ , \quad u_{\rm out} = \sigma_{\rm out}\, e^{\varrho_{\rm out}}\ ; \qquad \sigma_{\rm in} = (\pm)\,, \quad \sigma_{\rm out} = [\pm]$$

In terms of the ϱ coordinates, the Fourier trf. is more complicated.

The signs of $\sigma_{\rm in}$ and $\sigma_{\rm out}$ *do not commute*.

But the relation is now invariant under time shifts:

$$\varrho_{\rm in} \to \varrho_{\rm in} - \tau\ ; \quad \varrho_{\rm out} \to \varrho_{\rm out} + \tau\,.$$

So, plane waves transmute into plane waves. Outcome:

$$\sigma = \pm 1 \ , \qquad -\infty < \varrho < \infty:$$

$$\psi_{\rm in}(\varrho,\sigma) = \psi_\sigma^{\rm in}\, e^{-i\kappa(\varrho+\tau)} \ ; \qquad \psi_{\rm out}(\varrho,\sigma) = \psi_\sigma^{\rm out}\, e^{i\kappa(\varrho-\tau)}$$

The Fourier transformation gives at fixed ℓ, m

$$\boxed{\begin{pmatrix}\psi_+^{\rm out}\\ \psi_-^{\rm out}\end{pmatrix} = \frac{1}{\sqrt{2\pi}}\Gamma(\tfrac12 - i\kappa)\, e^{i\phi_\ell(\kappa)} \begin{pmatrix} e^{-\frac12\pi\kappa} & ie^{+\frac12\pi\kappa} \\ ie^{+\frac12\pi\kappa} & e^{-\frac12\pi\kappa}\end{pmatrix}\begin{pmatrix}\psi_+^{\rm in}\\ \psi_-^{\rm in}\end{pmatrix}}$$

Of course, the Fourier transform is unitary. Unitarity follows from:

$$|\Gamma(\tfrac12 - i\kappa)|^2 = \frac{\pi}{\cosh \pi\kappa}$$

Homework: Euler Gamma function:

If $\sigma = \pm 1$ then

$$\int_0^\infty \frac{\mathrm{d}z}{\sqrt{z}} e^{-\sigma z} z^{-i\kappa} = \Gamma(\tfrac{1}{2} - i\kappa) e^{\frac{-i\sigma\pi}{4} - \sigma\frac{\pi}{2}\kappa} .$$

$$\begin{pmatrix}\psi_+^{\rm out}\\ \psi_-^{\rm out}\end{pmatrix} = \frac{1}{\sqrt{2\pi}}\Gamma\left(\tfrac{1}{2} - i\kappa\right) e^{i\phi_\ell(\kappa)} \begin{pmatrix} e^{-\frac{1}{2}\pi\kappa} & ie^{+\frac{1}{2}\pi\kappa} \\ ie^{+\frac{1}{2}\pi\kappa} & e^{-\frac{1}{2}\pi\kappa}\end{pmatrix}\begin{pmatrix}\psi_+^{\rm in}\\ \psi_-^{\rm in}\end{pmatrix}$$

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To be read as follows: If κ is the new momentum variable, at each ℓ, m, we have a wave function $\psi_\sigma^{\rm in}(\kappa)$, and $\psi_\sigma^{\rm out}(\kappa)$ related by this unitary transformation matrix

the two signs, $\sigma = \pm 1$ mix.

This means that our particles fill region *I* as well as region *II* in the Penrose diagram.

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This BH universe has two asymptotic regions.

I and *II* *talk* to each other ! What does this mean?

The connection between regions *I* and *II* is called

the Einstein - Rosen bridge.

The ER bridge seems to connect *two* black holes.

To restore unitarity for a single black hole, we have only one option:

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To restore unitarity for a single black hole, we have only one option:

> region *II* describes the points on the horizon
> that are *antipodal* to the points in region *I*.

Sanchez-Whiting 1986/87

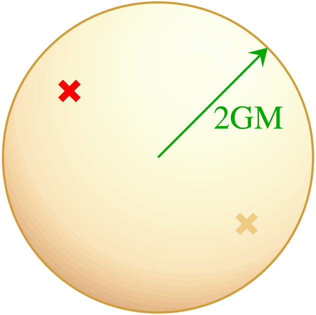
ℓ must be odd!

$$\begin{aligned} u^{\pm}(\theta, \varphi) &= -u^{\pm}(\pi - \theta,\, \varphi + \pi) \\ p^{\pm}(\qquad) &= -p^{\pm}(\qquad\qquad) \end{aligned}$$

only obeyed at odd ℓ:

$$Y_{\ell m}(\pi - \theta,\, \varphi + \pi) = (-1)^{\ell}\, Y_{\ell m}(\theta, \varphi)$$

If you could move faster than light, and you entered a black hole at one side,



you would re-emerge at the other side, CPT inverted!

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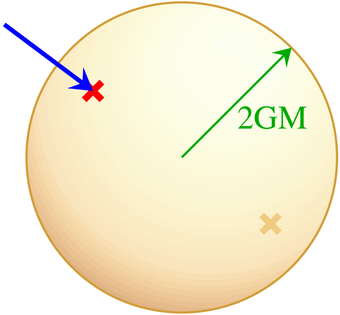

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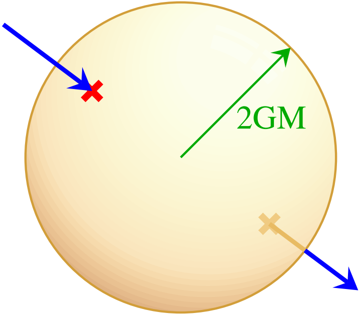

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Discussion.

We think that the apparent entanglement of regions I and II of the Penrose diagram is an important discovery. Before, we had been content with abstract functional integral expresions, which did not disclose so clearly the fact that one cannot ignore region II. Lesson learned: Whenever more explicit calculations are possible, we should do them; they yield much more understanding of what goes on.
We think that the Hawking-Perry-Strominger results using 'supertranslations' did not lead to answers as explicit as ours.

Yet our work is far from finished:

(1) We now have the microstates, and we can calculate the Hawking entropy, but a cut-off is needed limiting ℓ to a maximum value ($\ell < \mathcal{O}(R)$ in Planck units). How can we understand this cut-off? Some authors suggest that ours is "merely a semiclassical calculation", and that we ignored "quantum corrections" but we disagree, as the operators $p^-(\tilde{x})$ at different $\tilde{x}$ all commute. At small values of ℓ, our expressions should be very precise, but at ℓ close to the Planck length, we do expect deviations due to shifts arising from the transverse momenta.

(2) The representation of our algebra is different from Fock space, and therefore difficult to match with the Standard Model states. Trying to do this properly will be extremely important. It could lead to constraints on the SM coming from quantum gravity.

(3) Other forces between in- and out- particles can be considered: electro-magnetism and non-Abelian forces. At the functional integral level, this was done in Ref. (4). We could try to do this more explicitly now.

Note:

Antipodal identification of points on the horizon (*only* on the horizon, not in the bulk!) leads to *100% entanglement* of the Hawking particles at antipodal ponts, in principle an observable property.

We can calculate the BH microstates – except for the cut-off at $\ell \approx M_{\rm Planck}\, R_{BH}$. Taking discrete points (θ, φ) on the horizon, black holes have hair: one hair at every θ, φ, with end points on the tortoise coordinates: $u_{\rm out}$ describes exponentially growing hair, $u_{\rm in}$ describes exponentially shrinking hair, while

$\sigma_{\rm in}(\ell, m)$ and $\sigma_{\rm out}(\ell, m)$ form fermionic hair at fixed lengths.

Conclusion: On the scalp, there is a *fermionic degree of freedom*, the sign function σ_{in} or σ_{out}. These obey conservation laws, but σ_{in} and σ_{out} do not commute.

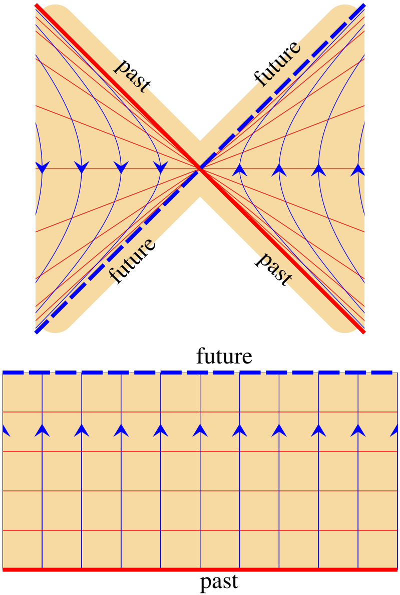
There is no black hole interior; region *II* of the Penrose diagram – with causality reflected in time – describes the antipodal part. When black hole forms, the interior region seems to be physical but of course it is invisible. Only when the black hole decays, one realises that the interior was never there.

Particles crossing the black hole interchange position with momentum and back.

The essential observation that the spherical partial waves of matter all return their quantum information independently, allows for new assessments of space-time properties that was not possible before.

All this could have been discovered decades ago.

String theory was not used – but may enter later –



Black Hole Formation

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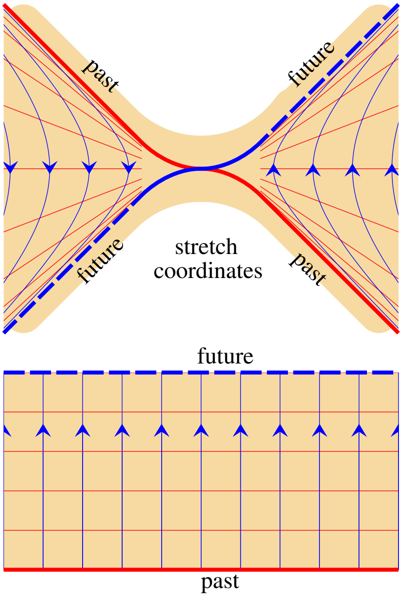

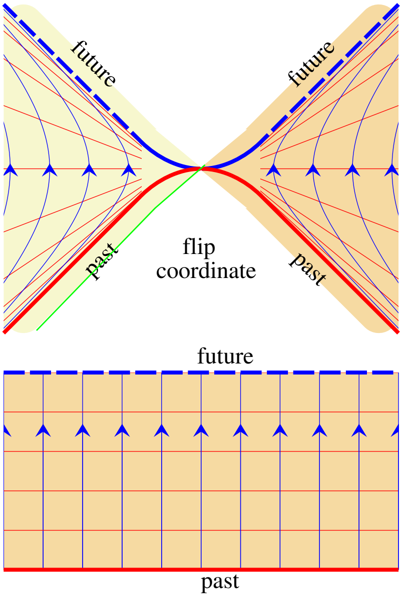

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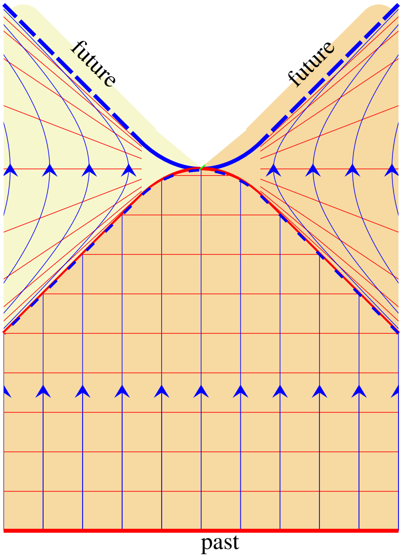

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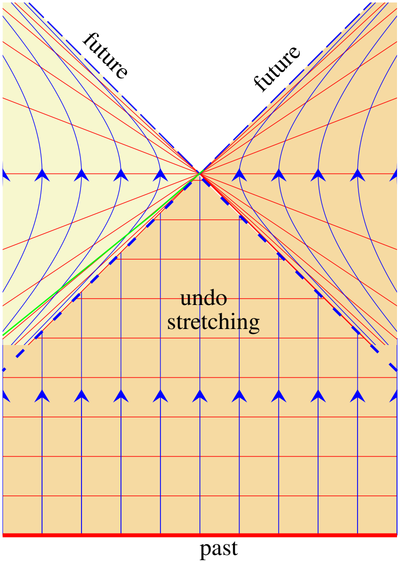

Black Hole Formation

References:

The importance of the gravitational shift (Shapiro delay) was already emphasized in:
(1) The gravitational shock wave of a massless particle (with T. Dray). Nucl. Phys. B253 (1985) 173
(2) Strings from gravity. Physica Scripta, Vol. T15 (1987) 143-150.
(3) The black hole interpretation of string theory. Nucl. Phys. B335 (1990) 138-154;
(4) The scattering matrix approach, J. Mod. Phys. A11 (1996) pp. 4623-4688, gr-qc/9607022.

Supertranslations:
(5) S.W. Hawking, M.J. Perry and A. Strominger: Soft Hair on Black Holes, arXiv: 1601.00921 [hep-th]

The partial wave expansion and antipodal entanglement:
(6) Diagonalizing the Black Hole Information Retrieval Process, arXiv:1509.01695
(7) Black hole unitarity and antipodal entanglement, arXiv:1601.03447[gr-qc] v3
(8) L. Mersini-Houghton, Entropy of the Information Retrieved from Black Holes, arXiv:1511.04795 [hep-th]
(9) The Quantum Black Hole as a Hydrogen Atom: Microstates without Strings Attached, arXiv:1605.05119