The complete $\mathcal{O}\left(\alpha_s^2\right)$ non-singlet heavy flavor corrections to DIS structure functions and sum rules

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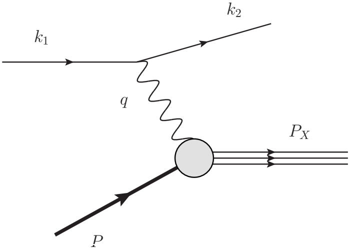
Deep-inelastic scattering

- Lepton-nucleon scattering experiment characterized by

$$
\begin{aligned}
Q^2 &= -(k_1 - k_2)^2 = -q^2, \\
x &= \frac{Q^2}{2P\cdot q} \\
Q^2 &\gg P^2,\ x \text{ fixed: Bjorken limit.}
\end{aligned}
\tag{1}
$$



- Leading-order cross section factorized into leptonic and hadronic tensors

$$
\frac{d\sigma}{dx\, dQ^2} \propto L_{\mu\nu}(k_1, q)\, W^{\mu\nu}(P, q) \tag{2}
$$

- $W^{\mu\nu}(P, q)$ is parameterized by structure functions: *e.g.* for unpolarized e.m. $e\,p$ scattering

$$
W_S^{\mu\nu} = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \frac{F_L(x, Q^2)}{2x} + \left[P_\mu P_\nu + \frac{q^\mu P^\nu + P^\mu q^\nu}{2x} - \frac{Q^2}{4x^2} g^{\mu\nu} \right] \frac{2x}{Q^2} F_2(x, Q^2) \tag{3}
$$

Factorization

- QCD radiative corrections to the structure functions are computed as convolution

$$F_i(x, Q^2) = x \int_x^1 \frac{d\xi}{\xi} C_i \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2} \right) q\left(\xi, \mu^2\right), \tag{4}$$

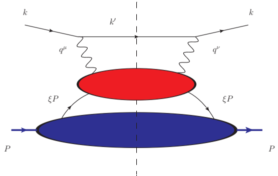
$C_i \left(x, \frac{Q^2}{\mu^2} \right)$: Wilson coefficients describing the *short distance* dynamics of the scattering

$$C_i \left(\xi, \frac{Q^2}{\mu^2} \right) = \sum_{n=0}^{\infty} \alpha_s^n \, c_i^{(n)} \left(\xi, \frac{Q^2}{\mu^2} \right). \tag{5}$$

$q(\xi, \mu^2)$: parton distribution functions, encoding the non-perturbative strong interaction.

- Heavy quark with mass m_Q enters $F_i(x, Q^2)$

$$F_i(x, Q^2, m_Q^2) = F_i^{\text{light}}(x, Q^2) + F_i^{\text{heavy}}(x, Q^2, m_Q^2), \tag{6}$$



- The precise measure of structure functions allows to extract α_s, m_c and PDFs.

Neutral current DIS

Cross section for e.m. scattering of polarized electron off polarized target: $W^{\mu\nu} = W_S^{\mu\nu} + i\, W_A^{\mu\nu}$,

$$W_A^{\mu\nu} = -\frac{M}{P\cdot q}\epsilon_{\mu\nu\rho\sigma} q^\rho \left[S^\sigma \widehat{g}_1(x,Q^2) + \left(S^\sigma - \frac{S\cdot q}{P\cdot q}P^\sigma \right)\widehat{g}_2(x,Q^2)\right], \tag{7}$$

M, P^μ and S^μ are respectively the nucleon mass, momentum and spin.

- Flavor non-singlet contributions (NS) are important in the difference $\left(\frac{d\sigma}{dxdQ^2}\right)^{\mathrm{ep}} - \left(\frac{d\sigma}{dxdQ^2}\right)^{\mathrm{en}}$

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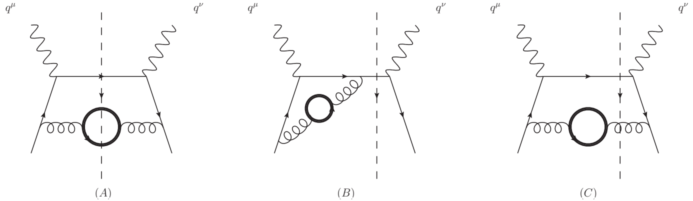
Factorization formula for NS contributions

$$\begin{aligned}\widehat{g}_1^{\text{NS}}(x,Q^2,m_Q^2) &= \frac{1}{2}\int_x^1 \frac{dz}{z}\left[C_{g_1,q}^{\text{NS}}\left(z,\frac{Q^2}{\mu^2}\right) + L_{g_1,q}^{\text{NS}}\left(z,\frac{Q^2}{m_Q^2},\mu^2\right)\right]\cdot\tilde{\Delta}\left(\frac{x}{z},\mu^2\right),\\ \tilde{\Delta}(x,\mu^2) &= \sum_i e_i^2\,\left[\Delta f_i(x,\mu^2) + \Delta f_{\bar{i}}(x,\mu^2)\right].\end{aligned} \tag{8}$$

- $C_{g_1,q}^{\text{NS}}$, massless Wilson coefficient, $\mathcal{O}(\alpha_s^3)$ terms computed (Moch,Vermaseren,Vogt '09)
- $L_{g_1,q}^{\text{NS}}$, massive Wilson coefficient, asymptotic limit $Q^2 \gg m_Q^2$ of $\mathcal{O}(\alpha_s^3)$ terms computed (Behring, Blümlein,De Freitas,von Manteuffel,Schneider '15)
- I will present the full $\mathcal{O}(\alpha_s^2)$ contribution to $L_{g_1,q}^{\text{NS}}$, including power corrections in $\frac{m_Q^2}{Q^2}$.

$L^{\rm NS}_{g_1,q}$: two-loop power corrections

Inclusive scattering process $q + \gamma^* \longrightarrow q + X$, heavy quarks in the final state X or in loops



$$L^{\rm NS}_{g_1,q}(z, Q^2, m_Q^2) = \underbrace{\Theta\left(\frac{\xi}{\xi+4} - z\right) L^{\rm NS,\,(R)}_{g_1,q}(z,\xi)}_{(A)} + \underbrace{\delta(1-z) L^{\rm NS,\,(V)}_{g_1,q}(\xi)}_{(B)}$$
$$\underbrace{- \left(\frac{\alpha_s}{4\pi}\right)^2 \beta_{0,Q} \log\left(\frac{m_Q^2}{\mu^2}\right)\left[\frac{1}{2} P^{(0)}_{qq} \log\left(\frac{Q^2}{\mu^2}\right) + c^{(1)}_{g_1,q}\right]}_{(C)}, \quad (9)$$

where $z = \frac{Q^2}{2p\cdot q}$, $\xi = \frac{Q^2}{m_Q^2}$, $\beta_{0,Q} = -\frac{4}{3} T_F$. $P^{(0)}_{qq}$ and $c^{(1)}_{g_1,q}$ are respectively the quark splitting function and the one-loop massless Wilson coefficient.

Real radiation

- Re-calculation of the Compton process

$$q + \gamma^* \to q + Q + \overline{Q}$$

The following variables are used

$$sq_1 = \sqrt{1 - \frac{4}{\xi}\frac{z}{1-z}},\ sq_2 = \sqrt{1 - \frac{4}{\xi}z},\ L_i = \log\left(\frac{1+sq_i}{1-sq_i}\right)\ \text{(i=1,2)},\ L_3 = \log\left(\frac{sq_2+sq_1}{sq_2-sq_1}\right),$$

$$\mathrm{di}_1 = \mathrm{Li}_2\left[(1-z)\frac{1+sq_1}{1+sq_2}\right],\ \mathrm{di}_2 = \mathrm{Li}_2\left(\frac{1-sq_2}{1+sq_1}\right),\ \mathrm{di}_3 = \mathrm{Li}_2\left(\frac{1-sq_1}{1+sq_2}\right),\ \mathrm{di}_4 = \mathrm{Li}_2\left(\frac{1+sq_1}{1+sq_2}\right).$$

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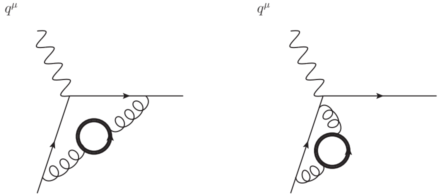
- Agreement with the literature (Buza, Matiounine, Smith, Migneron, van Neerven '96)

$$L^{\mathrm{NS,\,(R)}}_{g_1,q}(z,Q^2) = \left(\frac{\alpha_s}{4\pi}\right)^2 C_F T_F \Bigg\{ -\frac{8L_3\, sq_2}{9(z-1)\xi}\left(50z^3 - 11\xi + z(6\xi+20) - 2z^2(7\xi+12)\right)$$
$$+ \frac{2\, sq_1}{27(z-1)^2\xi}\left(1200z^4 + 265\xi - 4z^3(109\xi+490) + 2z^2(389\xi+618) - z(607\xi+466)\right)$$
$$+ \frac{4\, L_1}{3(z-1)^3\xi^2}\left(24z^4 - \xi^2 + 3z^2(\xi^2+6) - 2z^3(\xi^2+18)\right) + \frac{12z^3 - \xi^2 - z^2\xi^2}{3(z-1)\xi^2}$$
$$\times\left[4L_1L_2 + 8(-\mathrm{di}_1 + \mathrm{di}_2 + \mathrm{di}_3 - \mathrm{di}_4) - 4L_1\log\left(\frac{z^2}{1-z}\right)\right]\Bigg\}.$$

Virtual corrections



- The virtual corrections are given by

$$L^{\text{NS, (V)}}_{g_1,q}(\xi) = 2\mathcal{F}_1^{(2)}\left(-\frac{Q^2}{m_Q^2}\right),$$

where $\mathcal{F}_1^{(2)}$ come from the form factor diagrams.

Introducing the variable $\tilde{\lambda} = \sqrt{1-\frac{4}{\xi}}$ the result is

$$\begin{aligned} L^{\text{NS, (V)}}_{g_1,q}(\xi) = 2\left(\frac{\alpha_s}{4\pi}\right)^2 \mathrm{C}_F\mathrm{T}_F \Bigg\{ & \frac{3355}{81} - \frac{952}{9\xi} + \left(\frac{32}{\xi^2} - \frac{16}{3}\right)\zeta(3) \\ & + \left(\frac{440}{9\xi} - \frac{530}{27}\right)\log(\xi) + \tilde{\lambda}\left[\frac{184}{9\xi} - \frac{76}{9}\right]\left[\mathrm{Li}_2\left(\frac{\tilde{\lambda}+1}{\tilde{\lambda}-1}\right) - \mathrm{Li}_2\left(\frac{\tilde{\lambda}-1}{\tilde{\lambda}+1}\right)\right] \\ & + \left[\frac{8}{3} - \frac{16}{\xi^2}\right]\left[\mathrm{Li}_3\left(\frac{\tilde{\lambda}-1}{\tilde{\lambda}+1}\right) + \mathrm{Li}_3\left(\frac{\tilde{\lambda}+1}{\tilde{\lambda}-1}\right)\right]\Bigg\}. \end{aligned} \tag{10}$$

g_1 structure function

- The massive Wilson coefficient including charm and bottom quark contributions is

$$L^{\text{NS}}_{g_1,q}(z, Q^2, m_c^2, m_b^2) = L^{\text{NS}}_{g_1,q}(z, Q^2, m_c^2) + L^{\text{NS}}_{g_1,q}(z, Q^2, m_b^2). \quad (11)$$

  We computed the convolution (8) of $L^{\text{NS}}_{g_1,q}(z, Q^2, m_c^2, m_b^2)$ with PDFs[1].

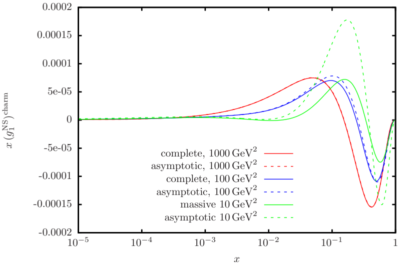


Figure: Comparison of the asymptotic and the exact charm quark contributions to $g_1^{\text{NS}}(x, Q^2)$.

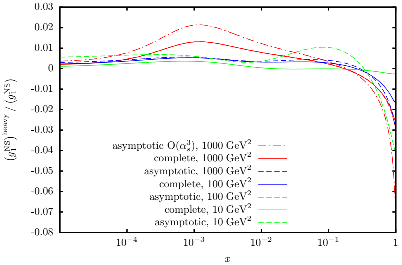


Figure: Impact of charm and bottom quarks on $g_1^{\text{NS}}(x, Q^2)$.

- Similarly, we computed the massive Wilson coefficients $L^{\text{NS},(2)}_{L,q}$, $L^{\text{NS},(2)}_{2,q}$ and computed $F_L(x, Q^2, m_Q^2)$, $F_2(x, Q^2, m_Q^2)$.

[1] These plots are done with BB09 POLPDF (polarized) and abm12_3_nnlo (unpolarized), $m_c = 1.59$ GeV and $m_b = 4.78$ GeV

Polarized Bjorken sum rule

The polarized Bjorken sum rule is the first moment of g_1^{NS}

$$\Delta g_1(Q^2) = \int_0^1 dx \left[g_1^{e\,p}(x, Q^2) - g_1^{e\,n}(x, Q^2) \right] = \overbrace{K_{g_1}(n_f)}^{\text{Parton model result}} \quad A^{g_1}(\alpha_s, Q^2) \tag{12}$$

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A^{g_1} corresponds to the first moment of the Wilson coefficients $C_{g_1,q}^{\text{NS}}$ and $L_{g_1,q}^{\text{NS}}$.

$$A^{g_1}(\xi) = 1 - \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-\frac{55}{12} + \frac{1}{3} n_f + C_{\text{pBj}}^{Q,(2)}(\xi) \right] + \mathcal{O}(\alpha_s^3, \alpha_s^4)$$

$$C_{\text{pBj}}^{\text{Q},(2)}(\xi) = -\frac{1}{16}\frac{\text{C}_F \text{T}_F}{315\xi^2} \left\{ 2100 \log\left(\frac{\lambda+1}{\lambda-1}\right)^2 - \xi\left(6\xi^2 + 2735\xi + 11724\right) + \lambda \log\left(\frac{\lambda+1}{\lambda-1}\right) \right.$$

$$\left. \times\, \xi\left(3\xi^3 + 106\xi^2 + 1054\xi + 4812\right) - \xi^2\left(3\xi^2 + 112\xi + 1260\right)\log(\xi) \right\}, \qquad \lambda = \sqrt{1 + \frac{4}{\xi}}$$

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$$A^{g_1}(\xi) = 1 - \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-\frac{55}{12} + \frac{1}{3} n_f + C^{Q,(2)}_{\mathrm{pBj}}(\xi) \right] + \mathcal{O}(\alpha_s^3, \alpha_s^4)$$

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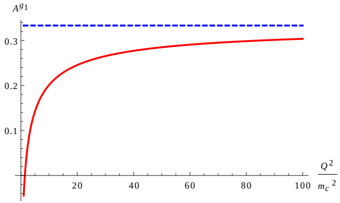

The **inclusive** definition of g_1^{NS} implies:

- Smooth transition $n_f \to n_f + 1$ at $Q^2 \gg m_c^2$. Logarithmic enhancements are not present.
- Negative contributions at $Q^2 \simeq m_c^2$.

Charged current DIS

Cross section for (anti)neutrino scattering off unpolarized target: $W^{\mu\nu} = W_S^{\mu\nu} + i\, W_A^{\mu\nu}$,

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$$F_{2,L}^{W^+-W^-} = 2\Big\{ x \int_x^1 \frac{dz}{z} \left[|V_{du}|^2\, d_v\left(\frac{x}{z}\right) - (|V_{du}|^2 + |V_{su}|^2)\, u_v\left(\frac{x}{z}\right) \right] \left(C_{2,L,q}^{\mathrm{NS}}(z) + L_{2,L,q}^{\mathrm{NS}}(z) \right),$$

$$+\tilde{x} \int_{\tilde{x}}^1 \frac{dz}{z} |V_{dc}|^2\, d_v\left(\frac{\tilde{x}}{z}\right) H_{2,L,q}^{\mathrm{NS}}(z) \Big\}, \qquad \tilde{x} = x\frac{Q^2 + m_Q^2}{Q^2}$$

$$F_3^{W^++W^-} = 2\Big\{ \int_x^1 \frac{dz}{z} \left[|V_{du}|^2\, d_v\left(\frac{x}{z}\right) + (|V_{du}|^2 + |V_{su}|^2)\, u_v\left(\frac{x}{z}\right) \right] \left(C_{3,q}^{\mathrm{NS}}(z) + L_{3,q}^{\mathrm{NS}}(z) \right)$$

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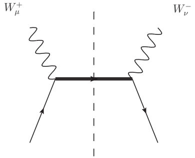
$$+\int_{\tilde{x}}^1 \frac{dz}{z}|V_{dc}|^2\, d_v\left(\frac{\tilde{x}}{z}\right) H_{3,q}^{\rm NS}(z)\Big\},$$



$H_{i,q}^{\rm NS}$ generated by the flavor-excitation process.
Note the CKM suppression $|V_{cd}|^2 \simeq 0.05$.

F_1 structure function

We implemented the numerical convolution of the necessary Wilson coefficients

- $H_i^{NS,(1)}$ (Gottschalk '81; Glück, Kretzer, Reya '96; Blümlein, Hasselhuhn, Kovacikova, Moch '11) and the asymptotic approximation of $H_i^{NS,(2)}$ (Blümlein,Pfoh,Hasselhuhn '14),
- $C_i^{NS,(1)}$, $C_i^{NS,(2)}$ (Zijlstra, van Neerven '92; Moch, Vermaseren '99; Moch, Rogal, Vogt '07),
- $L_{i,q}^{NS}$ for $i = 2, L$; $L_{3,q}^{NS} = L_{g_1,q}^{NS}$ are known from the neutral current calculation,

We construct the structure function $F_1(x, Q^2) = \frac{1}{2x}\left[F_2(x, Q^2) - F_L(x, Q^2)\right]$.

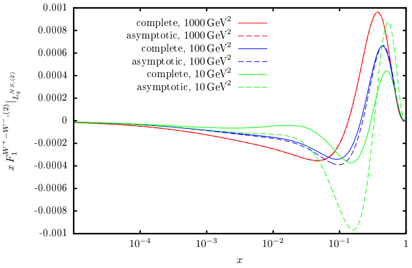


Figure: Exact and asymptotic $L_{1,q}^{NS}$ contribution to the structure function.

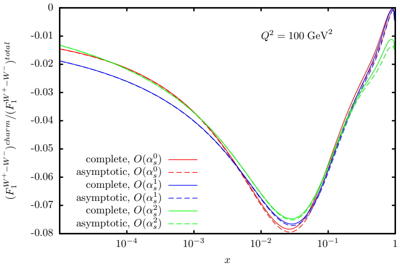


Figure: Relative charm quark contribution to the structure function.

The unpolarized Bjorken sum rule

The unpolarized Bjorken sum rule is derived from the first moment of F_1

$$\Delta F_1(Q^2) = \int_0^1 dx \left[F_1^{\overline{\nu}\, p}(x, Q^2) - F_1^{\nu\, p}(x, Q^2) \right] = K_1(n_f)\, A^{F_1}(\alpha_s, Q^2) \qquad (14)$$

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Massive contributions to A^{F_1} start at tree level, due to the H_1 Wilson coefficient.

$$\begin{aligned} A^{F_1} &= \left[1 - |V_{cd}|^2 \left(C_{\mathrm{uBj}}^{\mathrm{Q},(0)} - 1 \right) \right] - \left(\frac{\alpha_s}{\pi}\right) \left[\frac{2}{3} + |V_{cd}|^2 \left(C_{\mathrm{uBj}}^{\mathrm{Q},(1)}(\xi) + \frac{2}{3} \right) \right] \\ &\quad + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-\frac{23}{6} + \frac{8}{27}\, n_f + C_{\mathrm{uBj}}^{\mathrm{Q},(2)}(\xi) \right] + \mathcal{O}(\alpha_s^3), \\ C_{\mathrm{uBj}}^{\mathrm{Q},(2)}(\xi) &= \mathrm{C}_F \mathrm{T}_F \left\{ \frac{1129}{2520} - \frac{1}{2\xi^2} \log\left(\frac{\lambda+1}{\lambda-1}\right)^2 + \frac{107}{42\xi} - \frac{\xi}{420} + \lambda \log\left(\frac{\lambda+1}{\lambda-1}\right) \right. \\ &\quad \left. \times \left(-\frac{67}{420} - \frac{43}{42\xi} - \frac{\xi}{420} + \frac{\xi^2}{840} \right) + \left(\frac{1}{6} - \frac{\xi^2}{840} \right) \log(\xi) \right\} \end{aligned} \tag{15}$$

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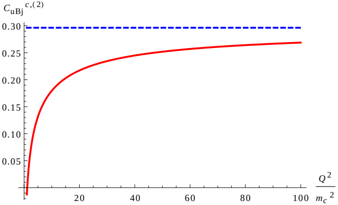
$$\begin{aligned} A^{F_1} &= \left[1 - |V_{cd}|^2 \left(C_{\text{uBj}}^{\text{Q},(0)} - 1\right)\right] - \left(\frac{\alpha_s}{\pi}\right) \left[\frac{2}{3} + |V_{cd}|^2 \left(C_{\text{uBj}}^{\text{Q},(1)}(\xi) + \frac{2}{3}\right)\right] \\ &\quad + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-\frac{23}{6} + \frac{8}{27} n_f + C_{\text{uBj}}^{\text{Q},(2)}(\xi)\right] + \mathcal{O}(\alpha_s^3), \\ C_{\text{uBj}}^{\text{Q},(2)}(\xi) &= \mathrm{C}_F \mathrm{T}_F \left\{ \frac{1129}{2520} - \frac{1}{2\xi^2} \log\left(\frac{\lambda+1}{\lambda-1}\right)^2 + \frac{107}{42\xi} - \frac{\xi}{420} + \lambda \log\left(\frac{\lambda+1}{\lambda-1}\right) \right. \\ &\quad \left. \times \left(-\frac{67}{420} - \frac{43}{42\xi} - \frac{\xi}{420} + \frac{\xi^2}{840}\right) + \left(\frac{1}{6} - \frac{\xi^2}{840}\right) \log(\xi) \right\} \end{aligned} \tag{15}$$



The typical featrues seen in A^{g_1} are recovered:

- absence of logarithmic terms and $n_f \to n_f + 1$ at $Q^2 \gg m_c^2$,
- negative contributions from virtual diagrams at $Q^2 \simeq m_c^2$.

higgstools

Summary and outlook

Neutral current DIS

- We calculate the Wilson coefficients $L^{\mathrm{NS}}_{g_1,q}$ exactly at two-loop order and compute the structure function g_1^{NS}: relevant deviations from the asymptotic results at $Q^2 \lesssim 10 GeV^2$.
- In a recent work, our group calculated also $L^{\mathrm{NS}}_{2,q}$, $L^{\mathrm{NS}}_{L,q}$, thus obtaining the heavy flavor corrections to all the non-singlet structure functions in neutral current DIS to $\mathcal{O}(\alpha_s^2)$.
- We study the effect of heavy flavors on the polarized Bjorken sum rule: the massless behaviour is recovered at high scales $Q^2 \gg m_c^2$, effects are smaller (and even negative) at intermediate scales $Q^2 \gtrsim m_c^2$.

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Outlook

The $\mathcal{O}(\alpha_s^2)$ calculation of heavy flavor effects will enter in the analysis of DIS data.

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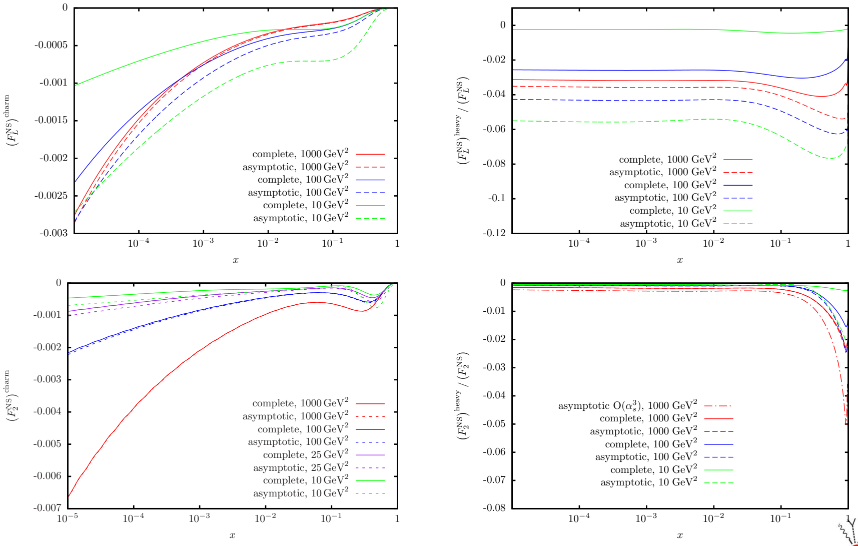
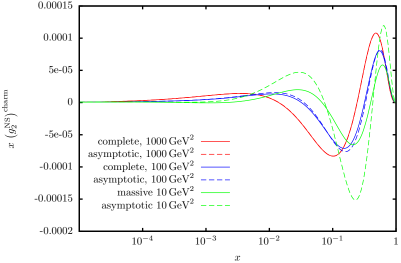
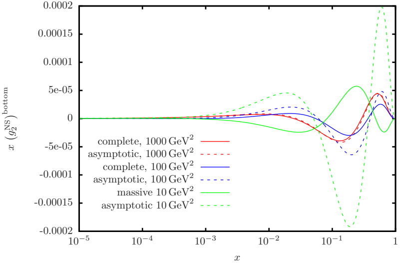
Thank you for your attention!

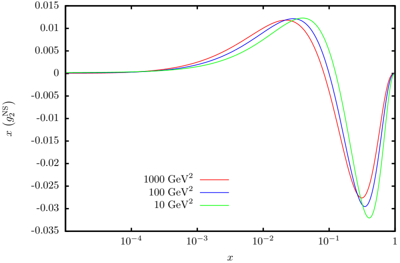
Neutral current plots: $F_L(x, Q^2)$ and $F_2(x, Q^2)$

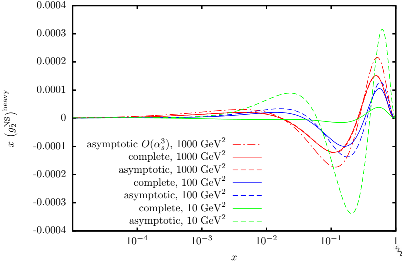


Neutral current plots: $g_2(x, Q^2)$









Charged current plots: $F_2(x, Q^2)$ and $F_3(x, Q^2)$