Conservation laws in the field theoretical formulation of gravity

Dmitrii GRAD

SPbSU

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Plan

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Conservation laws.

- In any physical theory conserved quantities are of special interest.
- Energy and momentum are the most important of them, because their conservation is related to homogeneity of the space-time.
- Using these conservation laws one can obtain some useful information about physical system without solving its equations of motion.

Stress-energy tensor.

The famous Noether's theorem gives us the formula of the stress-energy tensor $T^{\mu\nu}$, for which

$$\partial_\nu T^{\mu\nu} = 0,\ \mu,\nu = 0,1,2,3. \tag{1}$$

Gauss's theorem $\Rightarrow$ conserved integrals over the hypersurfaces $x^0 = \text{const}$:

$$P^\mu = \int d^3x\, T^{\mu 0} = \text{const}. \tag{2}$$

This is the energy-momentum vector, P^0 represents the energy and spatial components represent components of the momentum.

The problem of energy in general relativity.

In general relativity (for simplicity, without matter) one has

$$\partial_\nu \tau^{\mu\nu} = 0, \tag{3}$$

where $\tau^{\mu\nu}$ is the stress-energy "tensor" of gravitation.
The main trouble is that $\tau^{\mu\nu}$ is not a true tensor, so:

- Nonlocalizability of the energy density τ^{00}.
- Nonuniqueness of $\tau^{\mu\nu}$.

There are many pseudotensors: Einstein, Mϕller, Landau-Lifshitz, Komar etc.

The splitting theory.

The splitting theory is one of the field theoretical formulations of the theory of gravity:

- S. A. Paston, *Gravity as a field theory in flat space-time*, Theor. Math. Phys. 169(2), 2011, arXiv: 1111.1104 [gr-qc]

This theory considers a $(N-4)$-component real field $z^A(y)$ in N-dimensional ambient Minkowski space, $\quad A = 1, \dots, N-4$.

- Each field configuration corresponds to the some splitting of ambient space into a system of 4-dimensional surfaces $\mathfrak{S} : z^A(y) = \text{const}$.
- Surfaces don't interact and don't intersect.
- Any surface can be viewed as our spacetime.

Action of the splitting theory.

Action of the theory is the sum of the Einstein-Hilbert actions over all of the surfaces:

$$S = \int dz\, S_{\mathfrak{S}}(z) = \int dz d^4x \sqrt{-g} \left(-\frac{1}{2\varkappa} R \right), \qquad (4)$$

Changing the variables $\{z^A, x^\mu\} \to \{y^a\}$, we get

$$S = -\frac{1}{2\varkappa} \int dy \sqrt{|w|}\, R. \qquad (5)$$

This action gives the so-called Regge-Teitelboim equations for each surface

$$G^{cd} b^{a,}{}_{cd} = 0,\ a, c, d = 0, \dots, N-1. \qquad (6)$$

G^{cd} is the Einstein tensor, $b^{a,}{}_{cd}$ is the second fundamental form of the surface.

Profit.

- We have found the way to formulate gravity like a field theory in a flat spacetime.
- This theory is explicitly diffeomorphism-invariant.
- Such theory has no problems with the calculation of the stress-energy tensor, which will be a genuine tensor.

The canonical stress-energy tensor in the splitting theory.

The canonical (Noether) and the metrical (Hilbert) stress-energy tensors of the splitting theory were estimated.
The canonical one is

$$T_{\mathrm{N}}^{ab} = -\frac{\sqrt{|w|}}{\varkappa} G^{ab}. \tag{7}$$

It vanishes for any solution of the Einstein equations:

$$G^{ab} = 0 \Rightarrow T^{ab} = 0. \tag{8}$$

So, it is not so interesting.

The metrical stress-energy tensor in the splitting theory.

The metrical one is

$$T^{ab}_{\mathrm{H}} = -\frac{\sqrt{|w|}}{\varkappa} G^{ab} + \partial_g \Psi^{gab}, \tag{9}$$

where

$$\Psi^{gab} = \frac{\sqrt{|w|}}{\varkappa} \left[\left(\Pi^{ab} \Pi^{rs} - \Pi^{ar} \Pi^{bs} \right) b^{g,}{}_{rs} + \right. \\ \left. + \left(\Pi^{gr} \Pi^{bs} - \Pi^{ab} \Pi^{rs} \right) b^{a,}{}_{rs} \right] . \tag{10}$$

Π^{ab} is the projector onto the surface. Both of these stress-energy tensors are true tensors.

Therefore

- The energy density T^{00} is localizable.
- The energy $E = \int d\mathbf{y} T^{00}$ is explicitly diffeomorphism-invariant.

But these quantities are related to an unobservable field $z^A(y)$ in ambient space.

- What are they in terms of our customary 4-dimensional language?
- For instance, what will this energy be for the Schwarzschild solution?

Conclusions

- Gravity can be formulated as a field theory in a flat Minkowski space.
- True stress-energy tensors are present in this theory.
- So, we have the localizable energy density and the diffeomorphism-invariant energy.
- We need to translate these results into usual 4-dimensional language and to compare them with the standard general relativity results like the ADM energy.

Thank you for your attention!