Critical behavior of the systems with an antisymmetric tensor order parameter

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Microscopic action[1]:

$$S = \psi_i^+(\partial_t - \frac{\Delta}{2m} - \mu)\psi_i - \frac{\lambda}{2}(\psi_i^+\psi_i)(\psi_j^+\psi_j)$$

Using the Hubbard–Stratonovich transformation one can rewrite action as:

$$S = \psi_i^+(\partial_t - \frac{\Delta}{2m} - \mu)\psi_i + \chi_{ji}^+\chi_{ij} + \sqrt{\frac{\lambda}{2}}\chi_{ij}(\psi_i^+\psi_j^+) + \sqrt{\frac{\lambda}{2}}\chi_{ij}^+(\psi_i\psi_j)$$

From Schwinger equations:

$$\langle\chi_{ij}^+\rangle = \sqrt{\frac{\lambda}{2}}\langle\psi_i^+\psi_j^+\rangle; \quad \langle\chi_{ij}\rangle = \sqrt{\frac{\lambda}{2}}\langle\psi_i\psi_j\rangle$$

Effective action:

$$S(\chi) = tr(\chi^+(-\partial^2 + \tau)\chi) + \frac{g_1}{4}(tr(\chi\chi^+))^2 + \frac{g_2}{4}tr(\chi\chi^+\chi\chi^+)$$

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[1] Komarova, Nalimov, Honkonen; Theor.Math.Phys., **176**:1 (2013)
Kalagov, Kompaniets, Nalimov; Theor.Math.Phys., **181**:2 (2014)

Let's consider model of a real antisymmetric tensor field

$$\phi = \phi_{ik}(\mathbf{x});\ \phi_{ik} = -\phi_{ki};\ i,k = 1,...,n$$

in d-dimensional Euclidean space with the action:

$$S(\phi) = \frac{1}{2}tr(\phi(-\partial^2 + \tau_0)\phi) - \frac{g_{10}}{4!}(tr(\phi^2))^2 - \frac{g_{20}}{4!}tr(\phi^4)$$

The stability of the model require:

$2g_{10} + g_{20} > 0, \quad ng_{10} + g_{20} > 0$	for even values of n
$2g_{10}+g_{20} > 0, \quad (n-1)g_{10}+g_{20} > 0$	for odd values of n

$$S_R(\phi) = \frac{1}{2} tr(\phi(-Z_1\partial^2 + Z_2\tau)\phi) - \frac{g_1\mu^\varepsilon}{4!} Z_3 (tr(\phi^2))^2 - \frac{g_2\mu^\varepsilon}{4!} Z_4 tr(\phi^4)$$

This expression can be also obtained by multiplicative renormalization

$$\phi \to \phi Z_\phi, \quad \tau_0 \to \tau Z_\tau, \quad g_{01} \to g_1\mu^\varepsilon Z_{g_1}, \quad g_{02} \to g_2\mu^\varepsilon Z_{g_2}$$

Connection of the approaches:

$$Z_1 = Z_\phi^2, \quad Z_2 = Z_\tau Z_\phi^2, \quad Z_3 = Z_{g_1} Z_\phi^4, \quad Z_4 = Z_{g_2} Z_\phi^4$$

In frames of minimal subtractions (MS) renormalization scheme constants has the form of only poles in ε:

$$Z_i = 1 + \sum_{p=1}^{\infty} A_{ip}(g_{1,2})\, \varepsilon^{-p}$$

$$(\mathcal{D}_\mu + \beta_1 \partial_{g_1} + \beta_2 \partial_{g_2} - \gamma_\tau \mathcal{D}_\tau - n\gamma_\phi) W_n^R = 0$$

here $\mathcal{D}_x \equiv x\partial_x$, for any variable x, and RG functions are defined as follows:

$$\gamma_i \equiv \widetilde{\mathcal{D}}_\mu \ln Z_i; \quad \beta_i \equiv \widetilde{\mathcal{D}}_\mu g_i$$

$$\mathcal{D}_s \bar{g}_i(s, g) = \beta_i(\bar{g}), \quad \bar{g}_i(1, g) = g_i$$

Type of the fixed point is determined by the eigenvalues of the matrix:

$$\omega_{ik} = \partial \beta_i / \partial g_k |_{g=g_*}$$

Scaling behavior of Green functions described by critical exponents. For example:

$$\eta = 2\gamma_\phi^*$$

Index	Point A	Point B	Point C
g_1	0.537976	1.213492	1.025987
g_2	0	-2.190800	-1.367994
ω_1	1.459030	1.478211	1.712650
ω_2	-0.176972	-0.243158	-0.009264
η	0.023124	0.027542	0.023873

Table 1: Coordinates of fixed points and numerical values of corresponding exponents in dimension $d = 3$

Renormalized Green functions:

$$G_k(z, x_1, \ldots, x_k) = C^{-1} \int D\phi \; \phi(x_1) \ldots \phi(x_k) e^{S_R}$$

We are interested in behavior of coefficients only by number of [2] loops: $g_i \to z g_i$. Cauchy's integral formula:

$$G_k^N(z, x_1, \ldots, x_k) = \oint_\gamma dz \; \frac{G_k(z, x_1, \ldots, x_k)}{(-z)^{N+1}}$$

Let us stretch fields $\phi \to \sqrt{N}\phi$ and charges $g \to g/(N\mu^\varepsilon)$.

$$G_k^N(z, x_1, \ldots, x_k) \approx C^{-1} N^{k/2} \int D\phi \, dz \, \phi(x_1) \ldots \phi(x_k) e^{N(S - \ln(-z))}$$

[2] Kalagov, Kompaniets, Nalimov; Theor.Math.Phys., **181**:2 (2014)

Stationarity equations have the form:

$$-\partial^2\phi_{st} - \frac{z_{st}g_1}{3!}\phi_{st}tr(\phi_{st}^2) - \frac{z_{st}g_2}{3!}\phi_{st}\phi_{st}\phi_{st} = 0$$

$$\int d\mathbf{x}\{\frac{z_{st}g_1}{4!}(tr(\phi_{st}^2))^2 + \frac{z_{st}g_2}{4!}tr(\phi_{st}^4)\} = -1$$

Using transformations of the group $O(n)$ one get matrix of the field in the form:

$$\phi = \begin{pmatrix} s_1\sigma & 0 & \dots & 0 \\ 0 & s_2\sigma & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & s_p\sigma \end{pmatrix}; \quad \sigma = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$-\partial^2 s_i(\mathbf{x}) + \frac{zg_1}{3} s_i(\mathbf{x}) \sum_{j=1}^{p} s_j^2(\mathbf{x}) + \frac{zg_2}{6} s_i^3(\mathbf{x}) = 0$$

In case $d = 4$ solution of the system can be found in the form:

$$s_i(\mathbf{x}) = \frac{\alpha_i y}{|\mathbf{x} - \mathbf{x}_0|^2 + y^2},$$

where

$$\alpha_k^2 = \frac{-48}{(2kzg_1 + zg_2)}; \quad k = 1, \ldots, p$$

$$\alpha_m = 0; \quad m = p - k$$

and

$$z_k^{st} = -\frac{4k}{2kg_1 + g_2}$$

$$\beta_i^{(N)}(g_1, g_2) = C_i \cdot N! N^b (-a(g_1, g_2))^N (1 + O(\frac{1}{N}))$$

$$g_{1,2*}^{(N)} = Const \cdot N! N^{b+1} (-a(g_{1*}^{(1)}, g_{2*}^{(1)}))^N (1 + O(\frac{1}{N}))$$

$$\omega_{1,2}^{(N)} = Const \cdot N! N^{b+1} (-a(g_{1*}^{(1)}, g_{2*}^{(1)}))^N (1 + O(\frac{1}{N}))$$

$$\eta^{(N)} = Const \cdot N! N^b (-a(g_{1*}^{(1)}, g_{2*}^{(1)}))^N (1 + O(\frac{1}{N}))$$

Here $a_k(g_1, g_2) = -1/z_k^{st}$; $a(g_1, g_2) = max_k |a_k(g_1, g_2)|$; $b = ((n^2 - 2n)/2 + 11)/2$; and $g_{1,2*}^{(1)}$ are one-loop values of coordinates of fixed points.

$$f(z) = \sum_{N \geq 0} f_N z^N$$

Borel transformation is defined as:

$$F(t) = \sum_{N \geq 0} F_N t^N = \sum_{N \geq 0} \frac{f_N}{\Gamma(N + b_0 + 1)} \; t^N$$

Inverse Borel transformation:

$$f^{res}(z) = \int_0^\infty dt \; e^{-t} t^{b_o} F(zt)$$

One can construct analytic continuation using the mapping

$$u(t) = \frac{\sqrt{1 + at} - 1}{\sqrt{1 + at} + 1} \quad \Leftrightarrow \quad t(u) = \frac{4u}{a(u-1)^2}$$

Index	Point B	Point C
g_1	0.952345	0.896205
g_2	-1.335737	-1.194942
ω_1	0.721110	0.724327
ω_2	-0.008140	0.035955
η	0.003567	0.003546

Table 2: Numerical values of the critical indices in dimension $d = 3$ for fixed points **B** and **C**, obtained by the conformal-Borel summation method

$$S = tr((\partial + ieA)\chi^{+}(\partial - ieA)\chi) + \tau tr(\chi^{+}\chi) + \frac{1}{2}(\nabla\times A)^2 + \frac{1}{2\alpha}(\nabla A)^2 +$$

$$+ \frac{g_1}{4}(tr(\chi\chi^{+}))^2 + \frac{g_2}{4} tr(\chi\chi^{+}\chi\chi^{+})$$

There is 2 additional fixed points in this model, but they have real coordinates only for $n > 19$, when they appears to be saddle points.

Thank you for attention!