Hadronic Light-by-Light Scattering and Muon $g-2$: Dispersive Approach

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JHEP **09** (2015) 074 [arXiv:1506.01386 [hep-ph]]
JHEP **09** (2014) 091 [arXiv:1402.7081 [hep-ph]]

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15th June 2016

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Magnetic moment

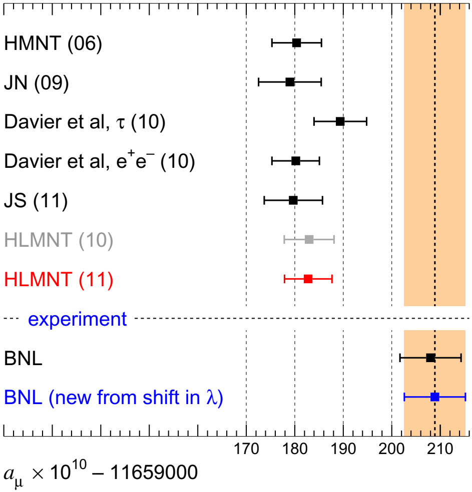
- relation of spin and magnetic moment of a lepton:

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell} \vec{s}$$

  g_ℓ: Landé factor, gyromagnetic ratio

- Dirac's prediction: $g_e = 2$
- anomalous magnetic moment: $a_\ell = (g_\ell - 2)/2$
- helped to establish QED and QFT as the framework for elementary particle physics
- today: probing not only QED but entire SM

$(g-2)_\mu$: comparison of theory and experiment

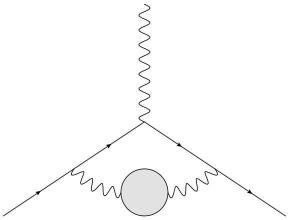


→ Hagiwara et al. 2012

$(g-2)_\mu$: theory vs. experiment

- discrepancy between SM and experiment $\sim 3\sigma$
- hint to new physics?
- new experiments (FNAL, J-PARC) aim at reducing the experimental error by a factor of 4
- theory error completely dominated by hadronic effects

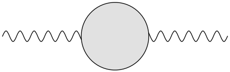
Hadronic vacuum polarisation: $\mathcal{O}(\alpha^2)$



- problem: QCD is non-perturbative at low energies
- first principle calculations (lattice QCD) may become competitive in the future
- current evaluations based on dispersion relations and data, can be systematically improved

Hadronic vacuum polarisation: $\mathcal{O}(\alpha^2)$

Photon hadronic vacuum polarisation function:

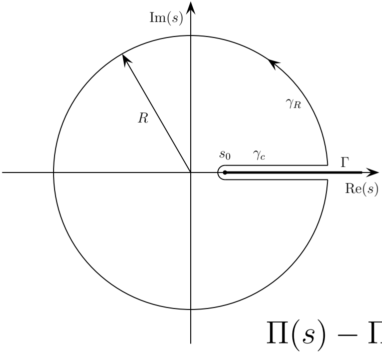

$$= -i(q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

Unitarity of the S-matrix implies the optical theorem:

$$\mathrm{Im}\Pi(s) = \frac{s}{e(s)^2}\sigma_{\mathrm{tot}}(e^+e^- \to \gamma^* \to \mathrm{hadrons})$$

Dispersion relation

Causality implies analyticity:



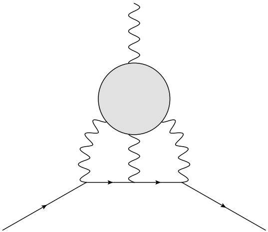
Cauchy integral formula:

$$\Pi(s) = \frac{1}{2\pi i} \oint_\gamma \frac{\Pi(s')}{s'-s} ds'$$

Deform integration path:

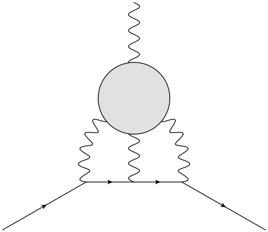
$$\Pi(s) - \Pi(0) = \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}\Pi(s')}{(s'-s-i\epsilon)s'} ds'$$

Hadronic light-by-light (HLbL) scattering



- up to now only model calculations
- uncertainty estimate based rather on consensus than on a systematic method
- will dominate theory error in a few years

Dispersive approach to HLbL



- make use of fundamental principles:
  - gauge invariance, crossing symmetry
  - unitarity, analyticity
- relate HLbL to experimentally accessible quantities

HLbL tensor: Lorentz decomposition

Solution for the Lorentz decomposition:

$$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma} \Pi_i(s, t, u; q_j^2)$$

- Lorentz structures manifestly gauge invariant
- scalar functions Π_i free of kinematic singularities
  $\Rightarrow$ ideal quantities for a dispersive treatment:
  use analyticity properties

Mandelstam representation

- we limit ourselves to intermediate states of at most two pions
- writing down a double-spectral (Mandelstam) representation allows us to split up the HLbL tensor:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \dots$$

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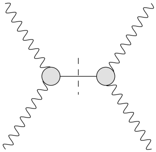
- one-pion intermediate state
- input: pion transition form factor



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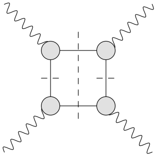
- two-pion intermediate state in both channels
- input: pion vector form factor



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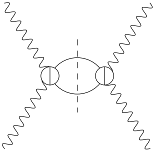
- two-pion intermediate state in first channel
- input: helicity partial waves for $\gamma^*\gamma^* \to \pi\pi$



Mandelstam representation

- we limit ourselves to intermediate states of at most two pions
- writing down a double-spectral (Mandelstam) representation allows us to split up the HLbL tensor:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi_{\mu\nu\lambda\sigma}^{\pi^0\text{-pole}} + \Pi_{\mu\nu\lambda\sigma}^{\text{box}} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \dots$$

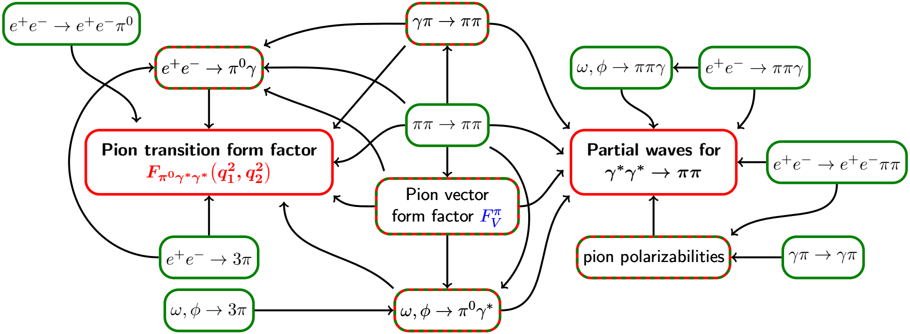
neglected so far: higher intermediate states

Summary

- our dispersive approach to HLbL scattering is based on fundamental principles:
  - gauge invariance, crossing symmetry
  - unitarity, analyticity
- we take into account the lowest intermediate states: π^0-pole and $\pi\pi$-cuts
- relation to experimentally accessible (or again with data dispersively reconstructed) quantities
- a step towards a model-independent calculation of a_μ

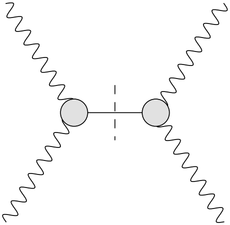
Backup

A roadmap for HLbL



→ Flowchart by M. Hoferichter

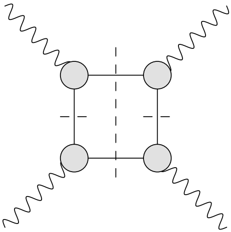
Pion pole



- input: doubly-virtual and singly-virtual pion transition form factors $\mathcal{F}_{\gamma^*\gamma^*\pi^0}$ and $\mathcal{F}_{\gamma^*\gamma\pi^0}$
- dispersive analysis of transition form factor:

  $\rightarrow$ Hoferichter et al., EPJC **74** (2014) 3180

Pion box



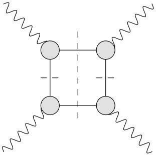
- simultaneous two-pion cuts in two channels
- Mandelstam representation explicitly constructed

$$\Pi_i = \frac{1}{\pi^2} \int ds' dt' \frac{\rho_i^{st}(s',t')}{(s'-s)(t'-t)} + (t \leftrightarrow u) + (s \leftrightarrow u)$$

- q^2-dependence: pion vector form factors $F_\pi^V(q_i^2)$ for each off-shell photon factor out

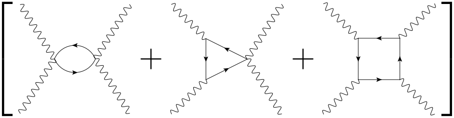
Pion box

- sQED loop projected on BTT basis fulfils the same Mandelstam representation
- only difference are factors of F_π^V
- $\Rightarrow$ box topologies are identical to FsQED:



$$\equiv F_\pi^V(q_1^2)F_\pi^V(q_2^2)F_\pi^V(q_3^2)$$

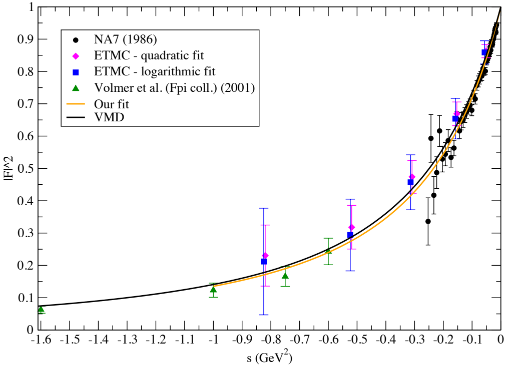
$\times$



- model-independent definition of pion loop

Pion box

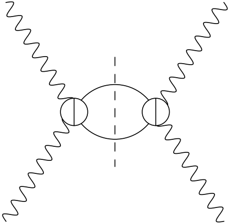
Pion vector form factor in the space-like region:



Preliminary results:

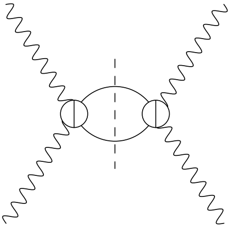
$$a_\mu^{\pi\text{-box}} = -15.9 \cdot 10^{-11}, \qquad a_\mu^{\pi\text{-box, VMD}} = -16.4 \cdot 10^{-11}$$

Rescattering contribution



- neglect left-hand cut due to multi-particle intermediate states in crossed channel
- two-pion cut in only one channel
- expansion into partial waves

Rescattering contribution



- unitarity relates it to the helicity amplitudes of the subprocess $\gamma^*\gamma^{(*)} \to \pi\pi$
- dispersive integrals over the imaginary parts allow the reconstruction of $\bar{\Pi}_{\mu\nu\lambda\sigma}$
- sum rules ensure cancellation of unphysical helicity amplitudes