neutrinos and the early universe$^1$

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first lecture

general overview

some non-equilibrium statistical physics

second lecture

evolution equations for dark matter and lepton and baryon asymmetries

example of a numerical solution
general overview
there is much neutrino oscillation data to respect\(^2\)

\[
V = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\
  s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23}
\end{pmatrix},
\]

where \(c_{ij} \equiv \cos \theta_{ij}\) and \(s_{ij} \equiv \sin \theta_{ij}\)

(NH) : \(\theta_{12} = 33.82^\circ + 0.78 \pm 0.76\), \(\theta_{23} = 49.6^\circ + 1.0 \pm 1.2\), \(\theta_{13} = 8.61^\circ + 0.13 \pm 0.13\), \(\Delta m_{21}^2 = 7.39 + 0.21 \pm 0.20 \times 10^{-5}\,\text{eV}^2\), \(\Delta m_{31}^2 = 2.525 + 0.033 \pm 0.032 \times 10^{-3}\,\text{eV}^2\)

(IH) : \(\theta_{12} = 33.82^\circ + 0.78 \pm 0.76\), \(\theta_{23} = 49.8^\circ + 1.0 \pm 1.1\), \(\theta_{13} = 8.65^\circ + 0.13 \pm 0.13\), \(\Delta m_{21}^2 = 7.39 + 0.21 \pm 0.20 \times 10^{-5}\,\text{eV}^2\), \(\Delta m_{23}^2 = 2.512 + 0.034 \pm 0.032 \times 10^{-3}\,\text{eV}^2\)

\(\Rightarrow\) determining \(\delta\) and mass ordering are among main future goals

\(^2\) e.g. I. Esteban et al, Global analysis of three-flavour neutrino oscillations: synergies and tensions in the determination of \(\theta_{23}, \delta_{CP}\), and the mass ordering, 1811.05487
let us explain these with a minimal model

$$\mathcal{L}_{\text{new-SM}} \equiv \mathcal{L}_{\text{old-SM}} + \bar{\nu}_R i \phi \nu_R$$

$$- (\bar{\nu}_R \phi \dagger h_\nu \ell_L + \bar{\ell}_L h_\nu \phi \nu_R)$$

$$- \frac{1}{2}(\bar{\nu}^c_R M_M \nu_R + \bar{\nu}_R M_M^\dagger \nu^c_R)$$

here $\nu_R$ is a new field, with three generations

singular value decomposition & field rotation

$$\Rightarrow M_M = \text{diag}(M_1, M_2, M_3), \text{ where } M_I \geq 0$$

we assume the $\{M_I\}$ to be set in increasing order
setting \( \tilde{\phi} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} \nu \\ 0 \end{pmatrix} \) we get a mass matrix

\[
M_D \equiv \frac{h^\dagger \nu}{\sqrt{2}} \Rightarrow -\mathcal{L}_{\text{new-SM}} \supset \bar{\nu}_L M_D \nu_R + \frac{1}{2} \bar{\nu}_R M_M \nu_R + \text{H.c.}
\]

inserting \(-1 = CC\) and noting that \(\nu_R^T C = \bar{\nu}_R^c\), we can write

\[
\bar{\nu}_L M_D \nu_R = \frac{1}{2} (\bar{\nu}_L M_D \nu_R - \nu_R^T M_D^T \bar{\nu}_L^T) = \frac{1}{2} (\bar{\nu}_L M_D \nu_R + \bar{\nu}_R^c M_D^T \nu_L^c)
\]

thereby the mass terms can be written as

\[
-\mathcal{L}_{\text{new-SM}} \supset \frac{1}{2} (\bar{\nu}_L \bar{\nu}_R^c) \begin{pmatrix} 0 & M_D \\ M_D^T & M_M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + \text{H.c.}
\]

\[
\equiv M_\nu
\]
extracting physical masses

the complex matrix $\mathcal{M}_\nu$ is symmetric and can be represented as$^3$

$$\mathcal{M}_\nu = U \, \text{diag}(m_\nu, M_h) \, U^T,$$

where $m_\nu$ and $M_h$ are real diagonal matrices with positive entries.

$\Rightarrow$ prediction: there are additional states, called sterile neutrinos

$\Rightarrow$ matrix $U$ implies mixing between active and sterile states

$^3$ this so-called takagi factorization ($\sim$ singular value decomposition) can most easily be implemented by treating the hermitean matrix $\mathcal{M}_\nu \mathcal{M}_\nu^\dagger$ via standard diagonalization
various orderings of $M_D$, $M_M$ are possible in principle

$M_M \ll M_D \Rightarrow$ dirac-like case, but too many states for cosmology\textsuperscript{4}

$M_M \sim M_D \Rightarrow$ fine-tuned case

$M_M \gg M_D \Rightarrow$ seesaw regime, consistent with all data

given that two mass differences $\Delta m^2_\nu$ are known, we restrict to a $2 \times 2$ matrix $M_M = \text{diag}(M_2, M_3)$ for explaining $m_\nu \& V$, keeping the third generation ($\equiv M_1$) as a “spectator”

notation: $\theta^2_{aI} \equiv \frac{|(M_D)_{aI}|^2}{M^2_I} \ll 1$

\textsuperscript{4} unless some states do not thermalize, cf. e.g. J. Lesgourgues and S. Pastor, \textit{Neutrino cosmology and Planck}, 1404.1740
the seesaw regime still leaves much freedom

if only one neutrino yukawa coupling contributes to a given mass difference, get the “seesaw” formula\(^5\)

\[ |\Delta m_\nu| \sim \frac{|(M_D)_{aI}|^2}{M_I} \]

traditionally: \(M_I^{\text{GUT?}} \sim 10^{15}\) GeV \(\Leftrightarrow h_\nu \sim 1\)

more recently: \(M_I \sim 1 \ldots 100\) GeV \(\Leftrightarrow h_\nu \sim 10^{-7} \ldots 10^{-6}\)

\(^5\) P. Minkowski, \(\mu \to e\gamma\) at a Rate of One Out of \(10^9\) Muon Decays?, PLB 67 (1977) 421; M. Gell-Mann, P. Ramond and R. Slansky, Complex Spinors and Unified Theories, 1306.4669; T. Yanagida, Horizontal Symmetry and Masses of Neutrinos, PTP 64 (1980) 1103
let us turn the tables and parametrize the ignorance \(^6\)

\[
M \equiv \begin{pmatrix}
M_2 & 0 \\
0 & M_3 \\
\end{pmatrix}, \quad R \equiv \begin{pmatrix}
\cos z & \sin z \\
-\sin z & \cos z \\
\end{pmatrix}, \quad z \in \mathbb{C},
\]

\[
P_{\text{NH}} \equiv \begin{pmatrix}
0 & e^{-i\phi_1} & 0 \\
0 & 0 & 1 \\
\end{pmatrix}, \quad P_{\text{IH}} \equiv \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{-i\phi_1} & 0 \\
\end{pmatrix}
\]

\[
\Rightarrow \quad h_\nu = -i\sqrt{M} \, R \, P \, \sqrt{m_\nu} \, V^\dagger \, \frac{\sqrt{2}}{v}
\]

---

the heavy states are unstable

$M_{2,3}$ can decay into $M_1$ and/or SM particles

if $M_1 \ll m_W$:

$$\Gamma_{N_1 \rightarrow \nu \nu \bar{\nu}} = \frac{G_F^2 M_1^5}{96\pi^3} \sum_{a=e,\mu,\tau} |\theta_{a1}|^2$$

partial decay width into photons:

$$\Gamma_{N_1 \rightarrow \nu \gamma} = \frac{9\alpha_{em} G_F^2 M_1^5}{256\pi^4} \sum_{a=e,\mu,\tau} |\theta_{a1}|^2$$
how do neutrinos behave in the early universe? a first guess

sterile states are exponentially rare when $T \ll M_I$

active states appear with Fermi distribution ($\omega_a \equiv \sqrt{k^2 + m_{\nu a}^2}$)

$$f(k) = n_F(\omega_a \pm \mu_L) = \frac{1}{\exp[(\omega_a \pm \mu_L)/T] + 1}$$

they contribute to the total energy and entropy densities, and thereby affect Big Bang Nucleosynthesis (BBN), Cosmic Microwave Background (CMB), Large Scale Structure (LSS), ...

but this works neither empirically nor conceptually!
for a realistic picture, need to solve an evolution equation

because \( T \) and thus \( n_F \) change, real processes are essential\(^7\)

\[
\partial_t f = -\Gamma(k) [f - n_F(\omega_a)] + \mathcal{O}[f - n_F(\omega_a)]^2
\]

\( \Gamma(k) = \) interaction rate from weak scatterings with the medium

equilibrium cannot be maintained if \( \Gamma(k) \) is small!

\(^7\) in covariant notation, \( \partial_t \rightarrow u \cdot \partial \), where \( u \) is the plasma four-velocity, but usually it is convenient to work in the plasma rest frame
this needs to be transcribed into an expanding background

friedmann equations \([a(t) \equiv \text{cosmological scale factor}]\)

\[
\frac{\dot{a}}{a} = \sqrt{\frac{8\pi}{3}} \frac{\sqrt{e}}{m_{Pl}} \equiv H,
\]

\[
d(e a^3) = -p \, d(a^3),
\]

where \(H = \text{hubble rate}, \ e = \text{energy density}, \ p = \text{pressure}\)

making use of \(de = Tds\) and \(e + p = Ts\), we find

\[
T \partial_t (s \, a^3) = 0
\]

which is known as the entropy conservation law
relation of time and temperature

\[ x \equiv \ln \left( \frac{T_{\text{max}}}{T} \right), \quad \mathcal{J} \equiv \frac{\text{d}x}{\text{d}t} = -\frac{\dot{T}}{T}, \]

\[ 0 = \frac{\partial_t (s a^3)}{s a^3} = \frac{\dot{T} s'}{s} + \frac{3 \dot{a}}{a} = -\frac{\mathcal{J}}{c_s^2} + 3H \]

where \( c_s^2 = s/(Ts') = \partial p/\partial e \) is the speed of sound squared

so we get \( \mathcal{J} = 3c_s^2 H \) and, with \( k_T \equiv k_{\text{min}} a(T_{\text{min}})/a(T) \),

\[
\frac{\text{d}f(x, k_T)}{\text{d}x} \approx -\hat{\Gamma}(k_T) \left[ f - n_F(\omega_T) \right], \quad \hat{\Gamma} \equiv \frac{\Gamma}{\mathcal{J}}
\]
so need microscopic rate ($\Gamma$) normalized to hubble rate ($\mathcal{J}$)

inverse propagator: $\mathcal{K} + \Sigma, \Sigma = \mathcal{K} \left( a + \frac{i\Gamma_K}{2} \right) + \psi \left( b + \frac{i\Gamma_u}{2} \right)$
solution of the evolution equation (ignoring masses)

\[ \hat{\Gamma}_u \gtrsim 1 \text{ until } x_1 \Rightarrow f(x_1, k) = \frac{1}{\exp(k/T_1)+1} \]

\[ \hat{\Gamma}_u \ll 1 \text{ afterwards } \Rightarrow f(x_2, k) = f(x_1, k \frac{a(T_2)}{a(T_1)}) \]

compute energy density in one chiral flavour of neutrinos:

\[ e_\nu = 2 \int_k k f(x_2, k) = 2 \int_k \frac{k}{\exp \left( \frac{k}{T_1 a(T_1)} \right) + 1} \]

\[ = 2 \left( \frac{T_1 a(T_1)}{a(T_2)} \right)^4 \frac{7 \pi^2}{8 \times 30} \]

\[ = \frac{7}{8} \left( \frac{T_1^3 a^3(T_1)}{T_2^3 a^3(T_2)} \right)^{\frac{4}{3}} \left( 2 \int_k \frac{k}{\exp(k/T_2) - 1} \right) \]
compare neutrino and photon energy densities

let $T_2 \ll 1$ MeV, so that electrons have become non-relativistic, but $T_2 \gg 1$ eV, so that neutrino masses are unimportant

because $sa^3$ is constant, the ratio is

$$\frac{T_1^3}{T_2^3} \frac{s(T_2)}{s(T_1)} = \frac{2}{2 + \frac{7}{8} \times 4} = \frac{4}{11}$$

$$\Rightarrow \quad \frac{e_\nu}{e_\gamma} \equiv \frac{7}{8} \left( \frac{4}{11} \right)^{\frac{4}{3}} N_{\text{eff}}$$

including masses, neutrino oscillations, smooth decoupling, etc:\(^8\)

$$N_{\text{eff}} \approx 3.045$$

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\(^8\) e.g. P.F. de Salas and S. Pastor, *Relic neutrino decoupling with flavour oscillations revisited*, 1606.06986
cosmological significance of active neutrinos

the energy density $e_\nu$ affects the hubble rate early times (down to photon decoupling)

$\Rightarrow$ CMB data yields $N_{\text{eff}} \simeq 2.96 \pm 0.34$

late times (temperature below $\nu$ masses): $e_\nu \propto \sum_a m_{\nu_a}$

$\Rightarrow$ CMB + LSS data yields $\sum_a m_{\nu_a} < 0.12...0.73$ eV

from extensive numerical simulations: $\Omega_\nu \simeq 0.3 \% \ll \Omega_{\text{dm}}$

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summary: active neutrino properties ⇔ cosmology

⇔ absolute mass scale can hopefully be constrained through a combination of upcoming data and extensive numerical simulations for the non-linear regime of LSS formation\(^\text{10}\)

⇒ CP violation and mass hierarchy of the active neutrinos influence the dynamics of the heavier sterile states, which themselves hopefully play a big role in cosmology

\(^\text{10}\) this is a “popular” enough challenge to be pursued e.g. by the flatiron institute
how about sterile neutrinos \( \equiv \) cosmology?

\[ 10^{-6} \leq \frac{M}{\text{GeV}} \leq 10^{12}, \quad 10^{-3} \leq \frac{T}{\text{GeV}} \leq 10^{12} \]

\( \nu_{\text{MSM}} \) 1\text{st} dark matter

\( \nu_{\text{MSM}} \) 2\text{nd} and 3\text{rd} leptogenesis & \( \Delta m_{\nu_a} \)

\( \Rightarrow \) nice playground for theory (model building and methods)
some non-equilibrium statistical physics
motivation

vacuum: $\langle \text{out} | \hat{T} \left\{ \exp \left[ -i \int_{-\infty}^{+\infty} dt \ H_{\text{int}}(t) \right] \right\} | \text{in} \rangle$

medium: no asymptotic states, but rather an instantaneous density matrix, whose time evolution we wish to track

$\Rightarrow$ what kind of “time ordering” is relevant?

$\Rightarrow$ upshot: in terms of a feynman prescription, need “retarded”$
\omega^2 - k^2 + i \ \text{sign}(\omega) 0^+ \text{ rather than the usual } \omega^2 - k^2 + i 0^+ \text{ rather than the usual} \omega^2 - k^2 + i 0^+$
general principle: interacting systems approach equilibrium\textsuperscript{11}

\[ \partial_t \langle O \rangle_{\text{non-eq}} = -\Gamma \left( \langle O \rangle_{\text{non-eq}} - \langle O \rangle_{\text{eq}} \right) \]

\[ + \mathcal{O} \left( \langle O \rangle_{\text{non-eq}} - \langle O \rangle_{\text{eq}} \right)^2 \]

the coefficient $\Gamma$ is independent of the non-equilibrium state of the system and is therefore determined by \textit{equilibrium} physics.

this approach is called “linear-response theory”, and is viable if $\Gamma \ll \omega_{\text{UV}}$, where $\omega_{\text{UV}} \sim M_I, \pi T$ represents fast time scales.

\textsuperscript{11} this statement is difficult to prove, but there is overwhelming practical evidence for it — as rare counterexamples so-called integrable systems can be mentioned (these include non-interacting systems), in which the dynamics is fully fixed by conservation laws.
consider a system described by a density matrix $\rho_{\text{full}}$

toy hamiltonian:

$$H = \omega_m a_m^\dagger a_m + J_m a_m + a_m^\dagger J_m, \quad \{a_m, a_n^\dagger\} = \delta_{mn}$$

schrödinger picture:

$$i\dot{\rho}_{\text{full}S} = [H, \rho_{\text{full}S}], \quad i\dot{O}_S = 0$$

heisenberg picture

$$i\dot{\rho}_{\text{full}H} = 0, \quad i\dot{O}_H = [O_H, H]$$

interaction picture:

$$i\dot{\rho}_{\text{full}I} = [H_{\text{int}I}, \rho_{\text{full}I}], \quad i\dot{O}_I = [O_I, H_0]$$
consider the density matrix “projected” onto a subsystem

definition:

\[ \rho_{ij}(t) \equiv \text{Tr}\{a_{iH}^\dagger(t)a_{jH}(t)\rho_{\text{fullH}}(t)\} \]

\[ = \text{Tr}\{a_{iI}^\dagger(t)a_{jI}(t)\rho_{\text{fullI}}(t)\} \]

equation of motion:

\[ i\dot{\rho}_{ij}(t) = (\omega_j - \omega_i)\rho_{ij}(t) \]

\[ + \text{Tr}\{[a_{iH}^\dagger(t)J_{jH}(t) - J_{iH}^\dagger(t)a_{jH}(t)]\rho_{\text{fullH}}(t)\} \]

\[ = (\omega_j - \omega_i)\rho_{ij}(t) \]

\[ + \text{Tr}\{[a_{iI}^\dagger(t)J_{jI}(t) - J_{iI}^\dagger(t)a_{jI}(t)]\rho_{\text{fullI}}(t)\} \]
insert time evolution of the full density matrix

represent time evolution of $\rho_{\text{full}I}$ through an integral equation

$$\rho_{\text{full}I}(t) = \rho_{\text{full}I}(0) - i \int_0^t dt' [H_{\text{int}I}(t'), \rho_{\text{full}I}(t')]$$

insert into evolution equation and assume "empty" initial state

$$\dot{\rho}_{ij}(t) = i(\omega_i - \omega_j)\rho_{ij}(t)$$

$$+ \int_0^t dt' \text{Tr} \left\{ [J^\dagger_{iI}(t)a_{jI}(t) - a^\dagger_{iI}(t)J_{jI}(t), J^\dagger_{mI}(t')a_{mI}(t') + a^\dagger_{mI}(t')J_{mI}(t')] \rho_{\text{full}I}(t') \right\}$$
omit fast oscillations, averaging to zero up to $O(\Gamma/\omega_{UV})$

insert $a_{mI}(t) = a_m e^{-i\omega_m t}$, $a_{mI}^\dagger(t) = a_m^\dagger e^{i\omega_m t}$, and keep only slow time dependences $\sim \omega_i - \omega_j$ where there is a cancellation

furthermore employ canonical anticommutators, to obtain

\[
\begin{align*}
[\ldots, \ldots] & \quad \Rightarrow \quad \delta_{im} e^{i\omega_m (t-t')} J_{mI}(t') J_{jI}(t) \\
& \quad - \quad a_{iI}^\dagger(t) a_{mI}(t') \{ J_{jI}(t), J_{mI}^\dagger(t') \} \\
& \quad + \quad \delta_{mj} e^{i\omega_m (t'-t)} J_{iI}^\dagger(t) J_{mI}(t') \\
& \quad - \quad a_{mI}^\dagger(t') a_{jI}(t) \{ J_{mI}(t'), J_{iI}^\dagger(t) \}
\end{align*}
\]
make again use of scale hierarchy $\Gamma \ll \omega_{\text{UV}}$

expand full density matrix in slow variations

$$\rho_{\text{full}I}(t') = \rho_{\text{full}I}(t) + (t' - t) \left( \frac{\partial}{\partial t} \rho_{\text{full}I}(t) + \cdots \right) \sim \frac{1}{\omega_{\text{UV}}} \sim \Gamma$$

recalling $a_{mI}(t') = a_{mI}(t)e^{i\omega_m(t-t')}$, recognize $\rho_{ij}$ on the rhs

\[
\dot{\rho}_{ij}(t) \approx i(\omega_i - \omega_j)\rho_{ij}(t) + \int_0^t dt' \{ e^{i\omega_m(t-t')} \left[ \delta_{im} \langle J_m(t') J_j(t) \rangle - \rho_{im}(t) \langle \{ J_j(t), J_m(t') \} \rangle \right] \\
&\quad \quad \quad e^{i\omega_m(t'-t')} \left[ \delta_{mj} \langle J_i(t) J_m(t') \rangle - \rho_{mj}(t) \langle \{ J_m(t'), J_i(t) \} \rangle \right] \} 
\]
identify equilibrium two-point correlators of $J, J^\dagger$

the anticommutators represent retarded/advanced correlators

$$\Pi_{ij}^R(\omega) = \lim_{t \to \infty} \int_{-t}^{t} dt' e^{i\omega(t-t')} i\theta(t - t') \langle \{J_i(t), J_j^\dagger(t')\} \rangle,$$

$$\Pi_{ij}^A(\omega) = \lim_{t \to \infty} \int_{-t}^{t} dt' e^{i\omega(t'-t)} (-i)\theta(t - t') \langle \{J_i(t'), J_j^\dagger(t)\} \rangle$$

if the ensemble is in equilibrium, the other terms (next to $\delta_{im}, \delta_{mj}$) can be related to $\Pi_{ij}^{R,A}$ through a “kubo-martin-schwinger relation”, which brings in $n_F(\omega - \mu) = \frac{1}{\exp((\omega - \mu)/T) + 1}$

⇒ explicit forms are shown later on
first lecture

general overview

some non-equilibrium statistical physics

second lecture

evolution equations for dark matter and lepton and baryon asymmetries

example of a numerical solution
evolution equations for dark matter and lepton and baryon asymmetries
predictions from the existence of $\nu_R$: there are heavy states and lepton asymmetries are violated

apart from $Y_R \simeq e_R/(M s)$, consider lepton asymmetry $Y_L \equiv n_L/s$, both of which are falling out of equilibrium:

$$Y_L' = - \hat{\Gamma}_L Y_L - \hat{\Gamma}_{L,R} (Y_R - Y_{eq})$$
$$Y_R' = - \hat{\Gamma}_R (Y_R - Y_{eq}) - \hat{\Gamma}_{R,L} Y_L$$

$\Rightarrow Y_L \neq 0$ can be generated if $Y_R \neq Y_{eq}$ and $\hat{\Gamma}_L$ is not huge

$\Rightarrow$ “sphaleron equilibrium”, $Y_B + Y_L \simeq 0$, then produces $Y_B$

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variables and coordinates for a precise treatment

- $Y_L \rightarrow$ flavour asymmetries $Y_a - \frac{1}{3} Y_B$, $a \in \{e, \mu, \tau\}$
- $Y_R \rightarrow$ density matrices $\rho_{IJ}(k, \pm)$

it is convenient to employ $\rho^\pm \equiv [\rho(k, +) \pm \rho(k, -)]/2$

cosmological evolution tracked through $x \equiv \ln(\frac{T_{\text{max}}}{T})$

redshift tracked through $k_T \equiv k(T_{\text{min}}) \frac{a(T_{\text{min}})}{a(T)}$

energies appear as $\omega_T \equiv \sqrt{k_T^2 + M_I^2}$

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original ideas were put forward by E.K. Akhmedov, V.A. Rubakov and A.Y. Smirnov, 
*Baryogenesis via neutrino oscillations*, hep-ph/9803255, and T. Asaka and M. Shaposhnikov, 
*The \( \nu \)MSM, dark matter and baryon asymmetry of the universe*, hep-ph/0505013; general 
formalism is similar to G. Sigl and G. Raffelt, *General kinetic description of relativistic mixed 
neutrinos*, NPB 406 (1993) 423, even if some terms were missed in that derivation
evolution equation for lepton asymmetries

\[ Y_a' - \frac{Y_B'}{3} = \frac{4}{s} \int_{kT} \text{Tr} \left\{ n_F' (\omega_T) \hat{A}^+_a \right\} \]
\[ + [ \rho^+ - n_F (\omega_T) ] \hat{B}^+_a \] (C-odd)
\[ + \rho^- \hat{B}^-_a \] (C-even)

1st term: washout term ("equilibration" viz. \( \hat{\Gamma}_L \)) \( \propto \{ \mu_a \} \)

2nd term: source from helicity-symmetric non-equilibrium

3rd term: source from helicity-asymmetric non-equilibrium
evolution equation for “dark matter”

\[(\rho^{\pm})' = -2 \hat{C}^{\pm} n_F'(\omega_T)
+ \{ \hat{D}^{\mp}, n_F(\omega_T) - \rho^+ \}
- \{ \hat{D}^{\mp}, \rho^- \}
+ i[\text{diag}(\hat{\omega}_T) - \hat{H}^+, \rho^\pm] - i[\hat{H}^-, \rho^{\mp}]\]

1st term: coupling to lepton asymmetries \( \propto \{ \mu_a \} \)

2nd & 3rd terms: equilibration rates (“collision integrals”)

4th term: corrections to dispersion relations (“matter effects”)

\[\text{36}\]
evolution equation for baryon asymmetry

schematically:

\[ Y'_a = \hat{F}_a + \frac{\hat{F}_{\text{anomaly}}}{6}, \quad Y'_B = \frac{\hat{F}_{\text{anomaly}}}{2} \]

\[ \Rightarrow \quad Y'_a - \frac{Y'_B}{3} = \hat{F}_a, \quad Y'_{B+L} = \sum_a \hat{F}_a + \hat{F}_{\text{anomaly}} \]

here \( \hat{F}_{\text{anomaly}} \) needs to be determined non-perturbatively,\(^{14}\) and switches from large to exponentially small at \( T \sim 130 \text{ GeV} \)

\[ \hat{F}_{\text{anomaly}} = -\frac{2n_G^2 \hat{\gamma}_{\text{anomaly}}(T)}{s} \frac{\tilde{\mu}_{B+L}}{T}, \quad n_G \equiv 3 \]

relation of chemical potentials and asymmetries

charge neutrality has to be respected and leads to a non-diagonal relation between asymmetries and chemical potentials\textsuperscript{15}

\[
\begin{pmatrix}
\tilde{\mu}_1 \\
\tilde{\mu}_2 \\
\tilde{\mu}_3 \\
\tilde{\mu}_{B+L}
\end{pmatrix}
= \frac{1}{144T^2}
\begin{pmatrix}
319 & 31 & 31 & -23 \\
31 & 319 & 31 & -23 \\
31 & 31 & 319 & -23 \\
-23 & -23 & -23 & 79
\end{pmatrix}
\begin{pmatrix}
n_1 - \frac{1}{3}n_B \\
n_2 - \frac{1}{3}n_B \\
n_3 - \frac{1}{3}n_B \\
n_B + \sum_a n_a
\end{pmatrix}
\]

\[\mathcal{O}(\alpha_w^{1/2})\]

\textsuperscript{15} e.g. S.Y. Khlebnikov and M.E. Shaposhnikov, *Melting of the Higgs vacuum: Conserved numbers at high temperature*, hep-ph/9607386; D. Bödeker and M. Sangel, *Order $g^2$ susceptibilities in the symmetric phase of the Standard Model*, 1501.03151; J. Ghiglieri and ML, *Precision study of GeV-scale resonant leptogenesis*, 1811.01971
coefficients arise from a retarded correlator

\[ \Pi^E_a(\tilde{K}) \equiv \int_X e^{i\tilde{K} \cdot X} \langle (\tilde{\phi}^\dagger \ell_a)(X)(\bar{\ell}_a \tilde{\phi})(0) \rangle \]

\[ \Pi^R_a(\mathcal{K}) \equiv \Pi^E_a(\tilde{K}) \bigg|_{k_n - i\mu_a \rightarrow -i[\omega + i0^+]} . \]

denoting by \( u_{k\tau I} \) an on-shell spinor of helicity \( \tau = \pm \), we need

\[ \frac{\bar{u}_{k\tau I} \text{ Im} \Pi^R_a(\mathcal{K}_I) u_{k\tau I}}{\omega_I} \equiv Q_{(a\tau)I}^{(\text{C-even})} + \bar{Q}_{(a\tau)I}^{(\text{C-odd})} \]

denoting \( Q_{(a)}^{\pm} \equiv [Q_{(a+)} \pm Q_{(a-)}]/2 \) yields

\[ A_{(a)II}^+ = \mu_a \text{ Re}(h_{Ia} h_{Ia}^*) Q_{(a)I}^+ \]
examples of direct processes contributing to $\text{Im} \Pi^R_{16}$

by optical theorem $\text{Im} A \Leftrightarrow A^* A$, e.g.

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examples of indirect processes contributing to $\Im\Pi^R_a$

here $\bullet$ indicates a higgs vev, i.e. active-sterile oscillation
example of a coefficient: \[ \Gamma_H \equiv \sum_{a,I=2,3} |h_{Ia}|^2 \langle Q_+(a)_I \rangle k_T \]
initial conditions for the evolution

⇒ assume that density matrices and all lepton asymmetries vanish at some $T_{\text{max}}$ which the universe reaches after inflation

⇒ at first not much goes on, but then there is complicated dynamics when first $\nu_R$ oscillations happen at around $T \sim T_{\text{osc}}$:

$$T_{\text{osc}} \sim 7 \times 10^4 \text{ GeV} \left(\frac{M}{\text{GeV}} \frac{|\Delta M|}{\text{MeV}} \frac{T}{\kappa}\right)^{1/3}$$
possibility of non-equilibrium expansion

consider a system with SM particles at temperature $T$, and other "heavy" particles, whose energy density and pressure are $e_H, p_H$

$$e = e_T + e_H, \quad p = p_T + p_H$$

starting again from Friedmann equations we find

$$T \partial_t (s_T a^3) = a^3(t) \sum_I \int_{k_T} 2 \omega_I \Gamma_{II} \left[ \rho_{II}^+(t, k_T) - n_F(\omega_I) \right]$$

this is called entropy release or dilution ($\sim dE = T dS$)

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\textsuperscript{17} e.g. R.J. Scherrer and M.S. Turner, Decaying particles do not “heat up” the Universe, PRD 31 (1985) 681
example of a numerical solution
a representative benchmark (\*) from a previous scan: \(^{18}\)

\[
M_2 = 0.7688 \text{ GeV} , \quad M_3 = 0.7776 \text{ GeV} , \\
z = 2.444 - i 3.285 , \\
\phi_1 = -1.857 , \quad \delta = -2.199 ,
\]

“inverted hierarchy”

not excluded, could be discovered, produces baryon asymmetry!

\(^{18}\) P. Hernández, M. Kekic, J. López-Pavón, J. Racker and J. Salvado, *Testable Baryogenesis in Seesaw Models*, 1606.06719; another extensive recent scan can be found in S. Eijima, M. Shaposhnikov and I. Timiryasov, *Parameter space of baryogenesis in the \(\nu\) MSM*, 1808.10833
a scan of degeneracies and yukawas

![Graph showing $Y_B$ and $Y_a$](https://example.com/graph.png)

**final $Y_B$**

**$Y_a$ at $T = 1$ GeV**

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19 J. Ghiglieri and ML, *Precision study of GeV-scale resonant leptogenesis*, 1811.01971
evolution of lepton and baryon asymmetries at high $T$

⇒ despite washout, a remnant lepton asymmetry is left over

⇒ the value is “large”: $|Y_a| \sim 3 \times 10^{-7} \approx 3 \times 10^3 Y_B$
the system settles into a “fixed point” at low $T$

⇒ helicity asymmetries balance against lepton asymmetries

⇒ flavour equilibrium even at $T \gg 10$ MeV
this could be interesting for dark matter production

physics of asymmetry-induced resonant conversion

for $M_1 \ll k_T$ the helicity-conserving indirect contribution reads

$$
Q(a-) + \overline{Q}(a-) \big|^{\text{indirect}} \approx \frac{v^2 M_1^2 \Gamma_u}{2\left[(M_1^2 + 2\omega_1 b)^2 + (\omega_1 \Gamma_u)^2\right]}
$$

at low $T$, $b = \tilde{b} \omega_1 + c$, where $\tilde{b}$ is positive, whereas $c$ is odd in the interchange $\mu_i \leftrightarrow -\mu_i$;\footnote{D. Nötzold and G. Raffelt, *Neutrino Dispersion at Finite Temperature and Density*, NPB 307 (1988) 924} for small $\omega_1 \Gamma_u$ this implies

$$
Q(a-) + \overline{Q}(a-) \big|^{\text{indirect}} \approx \frac{v^2 M_1^2}{2\omega_1} \pi \delta\left(2\tilde{b} \omega_1^2 - 2|c| \omega_1 + M_1^2\right)
$$
inspired by supposed detection, consider modern setup

1 light flavour ($M_1 \rightarrow \text{dm}$), 2 heavy flavours ($M_H \equiv M_{2,3} \rightarrow \Delta m_{\nu}$, $Y_B$), three lepton asymmetries, helicities

initial condition: maximal $|Y_a|$ from dynamics, not by hand

parameters of light flavour: $M_1 = 7$ keV, $\sin^2(2\theta) = 2 \times 10^{-10}$

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23 J. Ghiglieri and ML, Sterile neutrino dark matter via GeV-scale leptogenesis?, 1905.08814
there is entropy dilution from the decays of heavy flavours

⇒ because of the small yukawas, heavy flavours freeze out

⇒ after a while, they decay, releasing entropy
initial lepton asymmetries partly convert to dark matter

\[ \Omega_1 / \Omega_{dm} \]

\[ M_H = 0.8 \text{ GeV} \]

\[ \Omega_1 / \Omega_{dm} \]

\[ M_H = 0.8 \text{ GeV} \]

\[ \Omega_1 / \Omega_{dm} \]

⇒ entropy dilution is substantial for these parameters

⇒ final abundance \((\Omega_1 / \Omega_{dm} \equiv Y_{11}^+ / Y_{dm})\) remains below 10%
final result as a function of the initial asymmetry

\[ M_1 = 7 \text{ keV}, \sin^2(2\theta) = 2 \times 10^{-10} \]

\[ |Y_a| \text{ at } T \approx 5 \text{ GeV} \]

\[ \frac{\Omega_1}{\Omega_{\text{dm}}} \text{ at } T = 1 \text{ MeV} \]

\[ M_H = 0.2 \text{ GeV}, \quad M_H = 0.8 \text{ GeV}, \quad M_H = 4.0 \text{ GeV} \]

\[ \Rightarrow \] differences between \( M_H \) are due to entropy release

\[ \Rightarrow \] is \( |Y_a| \sim 10^3 \ldots 4 \) \( Y_B \) a glass half full or empty?
summary & what’s next

⇒ active neutrinos play a well-established role in cosmology
⇒ but they cannot serve as dark matter or seed baryogenesis
⇒ baryogenesis is possible with $\gtrsim 0.1$ GeV sterile neutrinos
⇒ theoretical uncertainties today at $\lesssim 50\%$ level
⇒ lepton asymmetries may be larger than baryon asymmetry
⇒ keV scale sterile neutrinos may constitute part of dark matter
⇒ this is a fascinating prospect, and merits further study