Protecting electroweak vacuum from New Physics destabilization

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Solving RG equations for the SM couplings, we obtain the RG improved effective potential:

$$V_{\text{eff}}(\phi) \sim \frac{1}{4} \lambda_{\text{eff}}(\phi) \phi^4$$

where $\lambda_{\text{eff}}(\phi)$ is the running coupling $\lambda(\mu)$ with $\mu = \phi$, obtained by running the system of RG equation of the SM couplings.
We have the formation of a second minimum $\phi_{min}^{(2)}$.

For the SM, where $M_H \sim 125\text{ GeV}$ and $M_t \sim 173\text{ GeV}$, this second minimum is lower than the EW vacuum $\Rightarrow$ We have to calculate the EW vacuum lifetime $\tau$. 
RG Improved Effective Potential

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\[ V_{\text{eff}}(\phi) \]

\[ \phi \]

\[ \text{EW} \]

\[ \text{New Minimum} \]
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Bounce solution in flat spacetime


- Euclidean action for a single component real scalar field \( \phi \):
  \[
  S[\phi] = \int d^4x \left[ \frac{1}{2} (\partial_\mu \phi)^2 + V(\phi) \right]
  \]
  where \( V(\phi) \) is a potential with a false vacuum \( \phi = \phi_{fv} \) and a true vacuum \( \phi = \phi_{tv} \).

- The bounce solution \( \phi_b(r) \) is a particular solution to the Euclidean Euler-Lagrange equation with \( O(4) \) symmetry. If \( r \) is the radial coordinate, the equation takes the form:
  \[
  \dddot{\phi}(r) + \frac{3}{r} \ddot{\phi}(r) = \frac{dV}{d\phi}
  \]
  The bounce solution is obtained imposing the boundary conditions:
  \[
  \phi(\infty) = \phi_{fv}, \quad \dot{\phi}(0) = 0
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Bounce solution in flat spacetime


- Euclidean action for a single component real scalar field $\phi$:

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- The bounce solution $\phi_b(r)$ is a particular solution to the Euclidean Euler-Lagrange equation with $O(4)$ symmetry. If $r$ is the radial coordinate, the equation takes the form:

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Bounce solution in SM - flat spacetime
Decay rate of the false vacuum

- Decay time of the false vacuum:

\[ \Gamma = \frac{1}{\tau} = De^{-\left(S[\phi_b] - S[\phi_{fv}]\right)} \equiv De^{-B} \]

where \( B = S[\phi_b] - S[\phi_{fv}] \) is called Tunneling Exponent, and the exponential of \(-B\) gives the “tree-level” contribution to the decay rate. Instead, \( D \) is the quantum fluctuation determinant.

- Good approximation to the prefactor \( D \). The EW vacuum tunneling time \( \tau = \Gamma^{-1} \) turns out to be:

\[ \tau \approx \left( \frac{R^4}{T^3_U} \right) e^{-B} \Rightarrow \tau_{flat} \sim 10^{639} T_U \]

where \( R \) is the size of the bounce, defined as the value of \( r \) such that \( \phi_b(R) = \frac{1}{2} \phi_b(0) \).
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- Good approximation to the prefactor \( D \). The EW vacuum tunneling time \( \tau = \Gamma^{-1} \) turns out to be:

\[ \tau \simeq \left( \frac{\mathcal{R}^4}{T_U^3} \right) e^{-B} \Rightarrow \tau_{\text{flat}} \sim 10^{639} T_U \]

where \( \mathcal{R} \) is the size of the bounce, defined as the value of \( r \) such that \( \phi_b(\mathcal{R}) = \frac{1}{2} \phi_b(0) \).
Bounce solution in curved spacetime


- Euclidean action for a single component real scalar field $\phi$:

$$S[\phi, g_{\mu\nu}] = \int d^4x \sqrt{g} \left[ -\frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right]$$

where $R$ is the Ricci scalar and $G$ is the Newton constant. Requiring again $O(4)$ symmetry, the (Euclidean) metric takes the form:

$$ds^2 = dr^2 + \rho^2(r)d\Omega_3^2$$

- The bounce solution is now given by $\phi_b(r)$ and $\rho_b(r)$, solutions of the coupled equations:

$$\ddot{\phi} + 3 \frac{\dot{\rho}}{\rho} \dot{\phi} = \frac{dV}{d\phi} \quad \dot{\rho}^2 = 1 + \frac{\kappa}{3} \rho^2 \left( \frac{1}{2} \dot{\phi}^2 - V(\phi) \right)$$

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Bounce solution in SM - curved spacetime

\[ \tau_{\text{flat}} \sim 10^{639} T_U \quad \Rightarrow \quad \tau_{\text{grav}} \sim 10^{661} T_U \]

Gravity tends to increase the tunneling time \( \tau \) respect to the flat spacetime background!
Bounce solution in SM - curved spacetime

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Gravity tends to increase the tunneling time \( \tau \) respect to the flat spacetime background!
Bounce solution in SM - curved spacetime

\[
\tau_{\text{flat}} \sim 10^{639} T_U \quad \Rightarrow \quad \tau_{\text{grav}} \sim 10^{661} T_U
\]

Gravity tends to increase the tunneling time \( \tau \) respect to the flat spacetime background!
Let’s add New Physics at the Planck scale

One way of parametrizing New Physics around $M_P$ is using the potential:

$$V(\phi) = V_{\text{eff}}(\phi) + \frac{\lambda_6}{6} \frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8} \frac{\phi^8}{M_P^4}$$

With $\lambda_6 < 0$ and $\lambda_8 > 0$ we describe a destabilizing New Physics.

Example with $\lambda_6 = -1.2$ and $\lambda_8 = 1$. 
Blue curve: bounce profile with $\lambda_6 = \lambda_8 = 0$, i.e. with SM alone.
Yellow curve: bounce profile with $\lambda_6 = -0.3$ and $\lambda_8 = 0.3$.
Green curve: bounce profile with $\lambda_6 = -0.01$ and $\lambda_8 = 0.01$. 

Bounce solution with New Physics - flat spacetime
Bounce solution with New Physics - curved spacetime

- **Blue curve**: bounce profile with \( \lambda_6 = \lambda_8 = 0 \), i.e. with SM alone.
- **Yellow curve**: bounce profile with \( \lambda_6 = -0.03 \) and \( \lambda_8 = 0.03 \).
- **Green curve**: bounce profile with \( \lambda_6 = -0.04 \) and \( \lambda_8 = 0.04 \).
Impact of New Physics on $\tau$

New bounce $\phi_{b}^{NP}(r) \Rightarrow$ New action $S[\phi_{b}^{NP}(r)] \Rightarrow$ New $\tau \sim e^{S[\phi_{b}^{NP}(r)]}$

<table>
<thead>
<tr>
<th>$\lambda_6$</th>
<th>$\lambda_8$</th>
<th>$\tau_{\text{flat}}/T_U$</th>
<th>$\tau_{\text{grav}}/T_U$</th>
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<tr>
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<td>0</td>
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<td>$10^{661}$</td>
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Gravity tends to stabilize the EW vacuum ($\tau_{\text{grav}}$ always higher than $\tau_{\text{flat}}$). However, New Physics has always a strong impact.

Stability problem in Standard Model
Computing the tunneling time
New Physics
Conclusions

Including New Physics
Non-minimal coupling to gravity

Stability diagram


100
Curved spacetime. Non-minimal coupling

A. Rajantie, S. Stopyra, PRD 95 (2017) 2, 025008

\[ S = \int d^4x \sqrt{-g} \left[ -\frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + V(\phi) + \frac{1}{2} \xi \phi^2 R \right] \]

Again $O(4)$ symmetry:

\[ \ddot{\phi} + 3 \frac{\dot{\rho}}{\rho} \dot{\phi} = \frac{dV}{d\phi} + \xi \phi R \quad \dot{\rho}^2 = 1 - \frac{\kappa}{3} \rho^2 \left( -\frac{1}{2} \dot{\phi}^2 + V(\phi) - 6 \xi \frac{\dot{\phi}}{\rho} \phi \dot{\phi} \right) \frac{1}{1 - \kappa \xi \phi^2} \]

with $R$ given by:

\[ R = \kappa \frac{\dot{\phi}^2(1 - 6\xi) + 4V(\phi) - 6\xi \phi dV/d\phi}{1 - \kappa \xi (1 - 6\xi) \phi^2} \]

For $\xi = 0$ these equations becomes the minimal coupling ones.
### New Physics vs. non-minimal coupling

Adding New Physics: $\lambda_6 = -1.2$ and $\lambda_8 = 1$

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$(\tau/T_U)_{SM}$</th>
<th>$(\tau/T_U)_{NP}$</th>
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<td>$0$</td>
<td>$10^{661}$</td>
<td>$10^{-58}$</td>
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<th>$(\tau/T_U)_{SM}$</th>
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<tr>
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<td>$10^{735}$</td>
<td>$10^{735}$</td>
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</tbody>
</table>

V. Branchina, E. Bentivegna, F. Contino, D. Zappalà, PRD 99 (2019) 9, 096029
The direct Higgs-gravity $\xi \phi^2 R$ provides a rescue against New Physics destabilization: a part a tiny range of values of $\xi$, we have always $\tau > T_U$.

For sufficiently large values of $\xi$ we have a washing out of New Physics effects: $\tau_{NP} \simeq \tau_{SM}$

New Physics vs. non-minimal coupling

\[ \xi = 0 \quad \xi = 1 \quad \xi = 10 \]

V. Branchina, E. Bentivegna, F. Contino, D. Zappalà, PRD 99 (2019) 9, 096029
In both cases, for the range of \( \lambda_6 \) and \( \lambda_8 \) showed, the EW vacuum is always stable \( \tau > T_U \), unlike the minimal coupling case \( \xi = 0 \).
The Higgs-gravity interaction term, whose presence is guaranteed by exceptionally well-known experimental facts (gravity, the Higgs boson, and the quantum nature of physical laws), acts as a universal stabilizing mechanism, that washes out any potentially destabilizing effect from high energy New Physics (for instance from unknown quantum gravity), protecting our universe from a disastrous decay.
Thank you for the attention!