Monopole Production in Heavy-Ion Collisions

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Based on work with Oliver Gould and Arttu Rajantie (ArXiv:1902.04388)
“In Search for the Unexpected”

- Everybody expected the Higgs Boson
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- Everybody hoped for supersymmetry
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Image: Monty Python, 1970

- ...but we still look anyway!
What are Magnetic Monopoles?

- Simplest description: at long distances
  \[ \vec{B} = \frac{g}{4\pi r^3} \vec{r} \]

- Permitted by standard model: see e.g. Dirac (1931), Cho & Maison (1996)


- Important consequence:
  \[ eg = 2\pi n \]

- Charge quantized
- Magnetic charge \( g \gg 1 \).

“One would be very surprised if the universe had made no use of it”
(Dirac 1931)
How to make monopoles? Schwinger pair production

- EM field unstable to decay into charged particle-antiparticle pairs (Schwinger 1951)
- Critical field strength $E \sim \frac{m_e^2}{e} \sim 10^{18} \text{ V m}^{-1}$. Currently unobserved but lasers are getting close!
- If monopoles exist, will be produced in a strong enough magnetic field; $B \sim \frac{m_M^2}{g}$
- Lack of observation of monopoles $\rightarrow$ lower bound on monopole masses
- Applies to all magnetically charged particles; not dependent on particular monopole model
Schwinger production of monopoles

- Affleck, Alvarez & Manton (1981):

\[ P \approx D \exp \left( -\frac{\pi m^2}{gB} + \frac{g^2}{4} \right) \]

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- Assumes constant magnetic field, zero electric field
- Highest field magnitudes in known universe are in heavy-ion collisions

**Question**

How does the temporal and spatial variation of the fields in heavy ion collisions affect pair production probability?
The Worldline Instanton Method

- Formulate Schwinger production as a vacuum decay process
- Using path integral manipulations and some approximations we can write the decay probability as a sum over closed worldlines in Euclidean space:

\[ P \sim \text{Im} \int \text{Closed worldlines} \mathcal{D}x \, e^{-S_E[A,A_{\text{ext}};x^\mu]/\hbar} \]

- In semiclassical limit \((S_E/\hbar \gg 1)\) this is dominated by stationary points of action — classical solutions to the equations of motion.
- Saddle points with one negative mode contribute to imaginary part

\[ P \sim e^{-S_{\text{saddle}}} \]
Electromagnetic fields in heavy-ion collisions

- Two heavy ($Z \gg 1$) ions fired towards each other at relativistic speeds


- Peak magnetic fields in peripheral collisions ($b = R$)
Electromagnetic fields in heavy-ion collisions

- We find that time-dependence is exponentially more important than spatial dependence

- Approximate analytical form:

\[ B_y(t, \vec{x} = 0, b = R) = \frac{B}{[1 + (\omega t)^2]^{3/2}} \]

- Dimensionless time-dependence parameter

\[ \xi := \frac{m\omega}{gB} \]
Calculating production rate in heavy-ion-like fields

- Find monopole paths in Euclidean space that are saddle-points of the EoMs
- Recall for constant fields

\[ \ln P \sim - \frac{\pi m^2}{gB} + \frac{g^2}{4} \]

- We find for heavy-ion-like fields to NLO in worldline self-interactions

\[ \ln P \sim - \frac{\pi m^2}{gB} \frac{4[E(-\xi^2) - K(-\xi^2)]}{\pi \xi^2} + \frac{g^2}{8} \left( \sqrt{1 + \xi^2} + \frac{1}{\sqrt{1 + \xi^2}} \right) \]

\( (\xi = \frac{m\omega}{gB}) \). We have computed all orders numerically (ArXiv:1902.04388)
Enhanced monopole production?

\[
\ln P \sim -\frac{\pi m^2}{gB} \frac{4\left[E(-\xi^2) - K(-\xi^2)\right]}{\pi \xi^2} + \frac{g^2}{8} \left( \frac{1}{\sqrt{1 + \xi^2}} + \frac{1}{\sqrt{1 + \xi^2}} \right) + \text{higher order terms}
\]

- Time dependence enhances production!
- Constant-field mass bound: \( m \gtrsim 40 \text{ GeV} \)
- Time-dependent mass bound: \( m \gtrsim 500 \text{ GeV} \)!
Parameters for real collisions

- Recall

\[ \xi = \frac{m\omega}{gB} \]

Putting in heavy-ion parameters

\[ \xi \approx \frac{2\pi mR}{Z_{\text{eg}}} = \frac{mR}{Zn} \]

- For any given ion species \( R \) and \( Z \) are fixed; we find

\[ \xi \approx \frac{m}{3n \text{GeV}} \]

- Independent of collision energy!

- For sufficiently high-mass monopoles (\( m > 3n \text{GeV} \)) we must take the effects of time-dependence into account at all relativistic collision energies.
Recap

Question
How does the temporal and spatial variation of the fields in heavy ion collisions affect pair production probability?

Partial Answer
- For monopoles with $m > 3n \text{GeV}$, time-dependence is crucial at all relativistic collision energies.
- If certain approximations are valid we can write down exponential dependence of pair production probability.
Breakdown of validity: point-like monopoles

- Worldline description assumes point-like monopoles — valid providing monopole length scales small compared to other length scales
  
  \[
  \frac{m}{gB\left(1 + \xi^2\right)^{3/2}} \gg \frac{g^2}{4\pi m}
  \]

- This is not satisfied in heavy-ion collisions at current collider energies (gets worse with higher energy)

- However, as all observed time-dependence effects enhance pair production, use of naive constant-field Schwinger result should proved a lower bound.
Overcoming this...

- Instanton calculation in field theory
- Rely on lattice techniques in theory describing monopole of interest
- Difficulty of finding saddle-point solution numerically
Summary

Question
How does the temporal and spatial variation of the fields in heavy ion collisions affect pair production probability?

Our Progress
- For exponential dependence of production rate need only consider time-dependence. Analytically tractable!
- Effect of time-dependence parametrised by $\xi \propto \frac{m}{n}$, independent of collision energy.

How to get to a full answer
- Need to take monopole structure into account to move beyond worldline approximation and compute instantons in full field theory. Watch this space!