d = 4 as the critical dimensionality of asymptotically safe interactions

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Motivation

- No direct experimental tests of fundamental properties of our universe
- Only consistency tests are possible

E.g., in String theory: $d = 10$ is the critical dimension of the superstring.

Asymptotic Safety: [Weinberg, 1979] Quantum properties of spacetime in terms of quantum fluctuations of the metric.
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\[
G_{\alpha}\quad \alpha_s
\]

Asymptotic Safety

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Standard Model matter + Asymptotically safe quantum gravity

\[ \Downarrow \]

Preferred dimensionality?
The $U(1)$ sector of the Standard Model

- Quantum fluctuations:
  vacuum is screening

$$k \partial_k g_Y(k) = \beta_{g_Y}|_{SM} = \frac{1}{16\pi^2} \frac{41}{6} g_Y^3$$

- Landau Pole: UV-completion
  requires new physics

  [Gell-Mann and Low, 1954], [Gockeler, Horsley, Linke, Rakow, Schierholz and Stuben, 1998],
  [Gies and Jaeckel, 2004]

$$g_Y^2(k) = \frac{g_Y^2(k_0)}{1 - \frac{1}{8\pi^2} \frac{41}{6} g_Y^2(k_0) \ln \left( \frac{k}{k_0} \right)}$$
The $U(1)$ sector of the Standard Model with AS quantum gravity

- Under inclusion of AS gravity:

\[
\beta_{g_Y} = -f g g_Y + \frac{1}{16\pi^2} \frac{41}{6} g_Y^3 + \mathcal{O}(g_Y)^4
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- Under inclusion of AS gravity:

$$\beta_{g_Y} = -f_g g_Y + \frac{1}{16\pi^2} \frac{41}{6} g_Y^3 + O(g_Y)^4$$

- FRG studies: $f_g > 0$ for $g > 0$

  (in $d = 4$)

  [Daum, Harst and Reuter, 2009], [Harst and Reuter, 2011],
  [Folkerts, Litim and Pawlowski, 2011], [Christiansen and
  Eichhorn, 2017], [Eichhorn and Versteegen, 2017],
  [Christiansen, Litim, Pawlowski and Reichert, 2017]
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  - Antiscreening effect of metric
    fluctuations
  - Abelian gauge coupling becomes
    asymptotically free

In $d = 4$, metric fluctuations might induce a (predictive) UV completion of the Abelian
gauge sector.  [Harst and Reuter, 2011], [Eichhorn and Versteegen, 2017], [Eichhorn, Held and Wetterich, 2017], [Eichhorn and Held, 2018]
Solution to the triviality problem in $d > 4$

- $[\bar{g}_Y] = \frac{4-d}{2}$
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- $[g_Y] = \frac{4 - d}{2}$

$$\beta_{g_Y} = g_Y \left( \frac{d - 4}{2} - f_g(d) \right) + \mathcal{O}(g_Y^3)$$
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\[ \beta_{g_Y} = g_Y \left( \frac{d-4}{2} - f_g(d) \right) + O(g_Y^3) \]

- Competition of $f_g(d)$ with canonical mass term.

- Necessary condition for UV completion:
  Effective dimensionality below four,  
  \[ f_g(d) > \frac{d-4}{2} \]
Specifically in asymptotically safe gravity:

- There exist indications that metric fluctuations must not be too strong.
- Interacting nature of gravity induces novel interactions in the matter sector.
- Beyond the weak-gravity regime, metric fluctuations can induce novel divergences in these interactions.

[Eichhorn and Gies, 2011], [Eichhorn, 2012], [Meibohm and Pawlowski, 2016], [Eichhorn, Held and Pawlowski, 2016],
[Christiansen and Eichhorn, 2017], [Eichhorn and Held, 2017], [Eichhorn, Lippoldt and Skinjar, 2017]
[Eichhorn, Lippoldt and MS, 2018]
Excluded strong gravity regime

- Evaluate \( f_g = -\frac{\eta A}{2} \) and "weak gravity bound" (FRG computations)
- Study explicitly conditions: \( f_g(d) > \frac{d-4}{2} \) in weak-gravity regime
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![Diagram showing UV-completion and excluded strong-gravity regime regions]
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- Study explicitly conditions: $f_g(d) > \frac{d-4}{2}$ in weak-gravity regime

Key Result

$U(1)$ gauge sector remains UV incomplete in $d \geq 6$, even in the presence of gravity.
Summary

- Strong enough metric fluctuations for gravitational solution of Landau pole.
- Effective dimensionality lowered below four.

\[ d > 4 \]

For Asymptotically safe gravity-matter models, \( d = 4 \) appears to be the only dimension to accommodate a UV-complete Abelian sector.

The predictive power of the asymptotic-safety paradigm could extend to fundamental parameters of the geometry.
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Thank you for your attention!
**Induced interaction**

- Example: Abelian gauge field $A_\mu$
  
  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

  - From kinetic term:
    
    $$S_{\text{kin}} = \frac{Z_A}{4} \int d^dx \sqrt{g} F_{\mu\nu} F^{\mu\nu}$$

  - Schematically:
    
    $$\beta_{w_2} = B_0(g) + w_2 B_1(g) + w_2^2 B_2$$

  - $\exists$ real FP only for $B_0 \leq \frac{B_1^2(g)}{4B_2}$

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Competing effects

\[ f_g(d) \sim \frac{d(d - 3)}{2} \frac{1}{\Gamma\left[\frac{d}{2}\right]} \]

- Increasing number of propagating gravitons
- Integration over momentum configurations

\[ \Rightarrow \text{For fixed } g, f_g(d) \text{ decreases with } d. \]

\[ \Rightarrow \text{UV completion requires growth of } g_*(d). \]
\[ \downarrow \text{Signals increasingly non-perturbative nature of gravity.} \]
Tool: Functional Renormalization Group

Non-Perturbative Renormalisation Group Equation [Wetterich, 1993], [Reuter, 1996]

\[ k \partial_k \Gamma_k = \frac{1}{2} \text{STr} \left( \left( \Gamma_k^{(2)} + R_k \right)^{-1} k \partial_k R_k \right) = \frac{1}{2} \]

\( \Gamma_k \) = scale dependent effective action
\( R_k \) = IR regulator

- exact 1-loop equation
- extract \( \beta \)-functions via projection
- truncation needed \( \rightarrow \) not closed

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Summary: Conditions for UV complete matter sector

- **UV completion for** $U(1)$ **sector**
  - Anti-screening contribution from metric fluctuations.
  - Condition:
    \[ f_g(d) > \frac{d-4}{2}. \]

- **UV completion for induced interactions**
  - Induced interactions can trigger new divergences
  - Condition:
    \[ B_0 \leq \frac{B_1^2(g)}{4B_2}. \]
Comparison of $f_g(d)$ with $f_{g,\text{crit}}(d)$

- **Direct gravitational contribution:**
  \[
  f_g = -\frac{\eta A}{2}\big|_{\text{grav}}
  \]

- **Diagrammatically:**
  
  ![Diagram](image)

  Reminder (with $f_{g,\text{crit}} = \frac{d-4}{2}$):
  \[
  \beta_g = g_Y (f_{g,\text{crit}} - f_g(d)) + \mathcal{O}(g_Y^3)
  \]

- **Opposite behavior of $f_{g,\text{crit}}(d)$ and $f_g(d)$**

- **Decrease of $f_g(d)$ has to be compensated by increasing $g$**

**Key Result**

Solution to the triviality problem shifts to more strongly coupled regime for $d > 4$. 

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Setup

- **Tool:** Non-Perturbative Renormalisation Group Equation
  
  \[ k \partial_k \Gamma_k = \frac{1}{2} \text{STr} \left( \left( \Gamma_k^{(2)} + R_k \right)^{-1} k \partial_k R_k \right) = \frac{1}{2} \]

- **Truncation of the full dynamics:**

  \[
  \Gamma_k = \frac{Z_A}{4} \int d^d x \sqrt{g} g^{\mu \rho} g^{\nu \kappa} F_{\mu \nu} F_{\rho \kappa} \\
  + \frac{w_2 k^{-d}}{8} \int d^d x \sqrt{g} (g^{\mu \nu} g^{\rho \lambda} F_{\mu \lambda} F_{\nu \rho})^2 \\
  - \frac{1}{16\pi} \frac{1}{g k^{-d+2}} \int d^d x \sqrt{g} (R - 2\lambda k^2) + S_{gf}.
  \]

- \( w_2, g, \lambda \): dimensionless couplings

- Gravitational dynamics: encoded in the values of \( g, \lambda \).
UV complete matter sector beyond $d = 4$?

- Area of allowed region for $g \in (0, 1000)$ and $\lambda \in (-1500, 0.5)$.
- Area shrinks to zero at $d_c \approx 5.8$.

- Calculation leading to green and red area:
  Subject to systematic errors due to truncation.
- Very large deformations necessary to make $d \geq 6$ viable (in explored range).
- Qualitative aspects of the scenario remain unchanged.