Quantum decoherence during inflation

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Introduction

- During inflation the universe rapidly expands and the observed classical distribution of inhomogeneities originates from the substantially non-classical state.

- The problem of transition from quantum to classical behavior can be solved in the context of the theory of decoherence induced by environment.

- We are taking into account that the long wavelength perturbations become unobservable and considering decoherence of background degrees of freedom while maintaining information about short wavelength perturbations.
The action of the scalar field, minimally coupled to gravity, has the form:

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa} R + \frac{1}{2} g^{\mu\nu} \partial_\nu \Phi \partial_\mu \Phi - V(\Phi) \right)$$

Let us consider inhomogeneous perturbations over a flat FLRW metric and homogeneous scalar field background:

$$ds^2 = e^{2\alpha(\eta)} \left\{ - (1 - 2A)d\eta^2 + 2(\partial_i B)dx^i d\eta + [(1 - 2\psi)\delta_{ij} + 2\partial_i \partial_j E + h_{ij}]dx^i dx^j \right\}$$

$$\Phi(\vec{x}, t) = \Phi(t) + \phi(\vec{x}, t)$$

Where $A, B, \psi, E$ — scalar functions, $h_{ij} = h_{ji}$, $h^i_i = 0$ and $\partial^i h_{ij} = 0$

After substituting perturbed metric and scalar field to the Hamiltonian, it decomposes into individual components:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_S + \mathcal{H}_T,$$
Hamiltonian

Introduce the following notation:

\[ \Phi_B(\vec{x}, \eta) = A + e^{-\alpha(\eta)} \left[ e^{\alpha(\eta)} (B - E') \right]' \] – Bardeen potential

\[ \phi^{(gi)}(\vec{x}, \eta) = \phi + \Phi'(B - E') \] – Gauge-invariant scalar

\[ v_S(\vec{x}, \eta) = e^{\alpha} \left[ \phi^{(gi)} + \Phi' \frac{\Phi_B}{\mathcal{H}} \right] \] – Mukhanov–Sasaki variable

\[ v^{(\lambda)}_{T,k}(\vec{x}, \eta) = \frac{e^{\alpha} h^{(\lambda)}_k}{\sqrt{2\kappa}} \]

\[ z = \frac{\partial \Phi}{\partial \eta} \frac{\partial \eta}{\partial \alpha} e^{\alpha} \]

\[ \omega^2_{S,k} = k^2 - \frac{z''}{z} \]

\[ \omega^2_{T,k} = k^2 - \frac{a''}{a} = k^2 - (\alpha')^2 - \alpha'' \]
then

\[ \hat{H}_0 = e^{-2\alpha} \left[ -\frac{\kappa}{12\mathcal{L}^3} \frac{\partial^2}{\partial \alpha^2} + \frac{1}{2\mathcal{L}^3} \frac{\partial^2}{\partial \Phi^2} + \mathcal{L}^3 V(\Phi) e^{6\alpha} \right] \]

\[ \hat{H}_S = \sum_k \hat{H}_{S,k} = \sum_k \left( -\frac{1}{2} \frac{\partial^2}{\partial v_{S,k}^2} + \frac{1}{2} v_{S,k}^2 \omega_{S,k}^2 \right) \]

\[ \hat{H}_T = \sum_\lambda \sum_k \hat{H}_{T,k} = \sum_{k,\lambda} \left( -\frac{1}{2} \frac{\partial^2}{\partial v_{T,\lambda}^2} + \frac{1}{2} v_{T,k}^2 \omega_{T,k}^2 \right) \]
Finally, master Wheeler-DeWitt equation is:

\[
\left( e^{-2\alpha} \left[ -\frac{\kappa}{12L^3} \frac{\partial^2}{\partial \alpha^2} + \frac{1}{2L^3} \frac{\partial^2}{\partial \Phi^2} + L^3 V(\Phi) e^{6\alpha} \right] + \right.
\]

\[
+ \sum_{k,S,T,\lambda} \left( -\frac{\partial^2}{\partial v_k^2} + v_k^2 \omega_k^2 \right) \right) \Psi_0(\alpha, \Phi) = 0
\]

The solution is sought in the form of the Born-Oppenheimer approximation:

\[
\Psi \left[ \alpha, \Phi, \{ v_{S,\vec{k}}, v_{T,\vec{k}}^{(\lambda)} \} \right] \approx \Psi_0 \prod_{\vec{k},\lambda} \Psi_{s,\vec{k}} \left( v_{S,\vec{k}} | \alpha, \Phi \right) \Psi_{T,\vec{k}} \left( v_{T,\vec{k}}^{(\lambda)} | \alpha, \Phi \right)
\]

For the homogeneous part we use the semiclassical WKB approximation:

\[
\Psi_0(\alpha, \Phi) \approx A(\alpha, \Phi) e^{iL^3 S_0(\alpha, \Phi) - L^3 \mathcal{R}(\alpha, \Phi)} , \ |\nabla \mathcal{R}| \ll |\nabla S_0|\]
We also introduce the conformal WKB time:

$$\frac{\partial}{\partial \eta} := e^{-2\alpha} \left[ -\frac{\pi}{6} \left( \frac{\partial S_0}{\partial \alpha} \right) \frac{\partial}{\partial \alpha} + \left( \frac{\partial S_0}{\partial \Phi} \right) \frac{\partial}{\partial \Phi} \right]$$

It goes along classical trajectories and in the first approximation it can be identified with the classical conformal time.

In terms of the WKB time, the Wheeler-DeWitt equation for the fluctuation part of $\Psi_k$ takes the form:

$$i \frac{\partial}{\partial \eta} \Psi_k(v_k|\alpha, \Phi) = \left[ -\frac{1}{2} \frac{\partial^2}{\partial v_k^2} + \frac{1}{2} \omega_k^2(\eta) v_k^2 \right] \Psi_k(v_k|\alpha, \Phi),$$

where $\omega_k^2(\eta)$ is calculated on the classical trajectory passing through $(\alpha, \Phi)$ in the direction given by $S_0$. 
The power-law inflation $e^\alpha \sim t^p$.

Slow-roll condition: $\dot{\Phi} \ll V(\Phi)$, $\ddot{\Phi} \ll 3H\dot{\Phi}$.

It can be parameterized by entering $\varepsilon = -\frac{\dot{H}}{H^2}$, $\delta = -\frac{\ddot{\Phi}}{H\dot{\Phi}}$. In this case it turns out that $\varepsilon = \delta = \frac{1}{p} = \text{const}$. And if we go to the conformal time $e^\alpha \sim \eta^{1+\beta}$.

Then frequencies in Mukhanov-Sasaki equation are

$$\omega_{S,k}^2 = \omega_{T,k}^2 = \vec{k}^2 - \frac{2 + 3\varepsilon}{\eta^2} = \vec{k}^2 - \frac{\beta(1 + \beta)}{\eta^2}, \quad \text{where} \ (1 + \beta) = \frac{p}{1 + p}$$

Then the equation for classical fluctuations is

$$\ddot{f}_\vec{k} + \left(\vec{k}^2 - \frac{\beta(1 - \beta)}{\eta^2}\right)f_\vec{k} = 0$$

This is Bessel equation, and its solution can be written in terms of the Hankel functions as:

$$f_\vec{k} = C_1 \sqrt{\eta} H^{(1)}_{\beta + \frac{1}{2}}(k\eta) + C_2 \sqrt{\eta} H^{(2)}_{\beta + \frac{1}{2}}(k\eta)$$
Calculation of the wave function for fluctuations

We choose following solution

\[
\begin{align*}
\vec{k} &= \frac{\sqrt{\pi}}{2} \sqrt{\eta} H^{(1)}_{\beta + \frac{1}{2}} (k \eta) \\
\vec{f}^* &= \frac{\sqrt{\pi}}{2} \sqrt{\eta} H^{(2)}_{\beta + \frac{1}{2}} (k \eta)
\end{align*}
\]

it satisfies

\[
\begin{align*}
W_f = \dot{f}^* f - f^* \dot{f} &= i \\
f \xrightarrow{\eta \to -\infty} \frac{e^{-i k \eta}}{\sqrt{2k}}
\end{align*}
\]

Vacuum solution:

\[
\Psi_{\vec{k}, 0} (v_k) = \frac{1}{(2\pi)^{1/4}} \frac{1}{\sqrt{f^*}} e^{\frac{i}{2} \frac{\dot{f}^*}{f^*} v_k^2}
\]

The wave function of fluctuations as a WKB package made of a bunch of classical trajectories:

\[
\Psi \big|_{S_0=\text{const}} = A(\beta) \prod_{\vec{k}} \Psi_{S, \vec{k}} (v_{S, \vec{k}} | \beta) \Psi_{T, \vec{k}} (v_{T, \vec{k}} | \beta) =
\]

\[
= A(\beta) \prod_{\vec{k}} \frac{1}{(2\pi)^{1/2}} \frac{1}{f^*} e^{\frac{i}{2} \frac{\dot{f}^*}{f^*} \left( v_{S, k}^2 + v_{T, k}^2 + v_{(\times) k}^2 \right)}
\]
Obtaining a reduced density matrix for short wavelength modes

Corresponding density matrix:

$$\rho(\beta, \tilde{\beta}, \{v_{s,k}, v_{T,k}\}, \{\tilde{v}_{s,k}, \tilde{v}_{T,k}\}) = \prod_{\tilde{k}, \tilde{p}} A(\beta) A^*(\tilde{\beta}) \psi_{\tilde{k}}(\beta, v_{\tilde{k}}) \psi^*_{\tilde{p}}(\tilde{\beta}, v_{\tilde{p}}) =$$

$$= A(\beta) A^*(\tilde{\beta}) \prod_{\tilde{k}, \tilde{p}} \frac{1}{2\pi} \frac{1}{f^*(\beta) f(\tilde{\beta})} e^{\frac{i}{2} \frac{f^*(\beta)}{f(\beta)} \left( v_{S,k}^2 + v_{(+)}^2 + v_{T,k}^2 \right) - \frac{i}{2} f(\beta) \left( v_{S,p}^2 + v_{(+)}^2 + v_{T,p}^2 \right)}$$

The reduced density matrix for short wavelength modes

$$\rho^{red} = A(\beta) A^*(\tilde{\beta}) \prod_{k < \frac{1}{|\eta|}} \left( \frac{i}{\dot{f}^* (\beta) f(\tilde{\beta}) - \dot{\tilde{f}} (\tilde{\beta}) f^* (\beta)} \right)^{\frac{3}{2}} \times$$

$$\times \prod_{k > \frac{1}{|\eta|}} \frac{1}{2\pi} \frac{1}{f(\tilde{\beta}) f^* (\beta)} e^{-\frac{i}{2} \left( \frac{\dot{f} - \dot{f}^*}{f f^*} \right) \left( v_{S,k}^2 + v_{(+)}^2 + v_{T,k}^2 \right) \frac{1}{2}}$$
Transition to the continuous spectrum of the momenta

We replace the product with the exponent of the sum of logarithms:

\[
\prod_{k < \frac{1}{|\eta|}} \left( \frac{i}{f^*(\beta)f(\tilde{\beta}) - \dot{f}(\tilde{\beta})f^*(\beta)} \right)^{\frac{3}{2}} = e^{\sum_{k < \frac{1}{|\eta|}} \frac{3}{2} \ln \left( \frac{i}{f^*(\beta)f(\tilde{\beta}) - \dot{f}(\tilde{\beta})f^*(\beta)} \right)} = e^{\sum_{k < \frac{1}{|\eta|}} -\frac{3}{2} \ln \left( (-i)f^*(\beta)f(\tilde{\beta}) - \dot{f}(\tilde{\beta})f^*(\beta) \right)}
\]

The sum in the exponent can be replaced back by the integral

\[
\sum_{k < \frac{1}{|\eta|}} \{\ldots\} \rightarrow \mathcal{L}^3 \int_{\frac{1}{\eta}}^{0} d^3k \{\ldots\}
\]
The result of numerical integration

\[ \eta = -10 \]

\[ \eta = -1 \]

\[ \eta = -0.1 \]
• We considered the evolution of the universe, which was originally in a vacuum state.

• Over time, an increasing number of perturbation modes of the metric and inflaton field goes beyond the cosmological horizon and becomes unobservable. We showed that during this process, the density matrix becomes mixed for short wavelength modes, in the limit diagonal in the $\beta$, the variable characterizing the rate of inflation.

• This suggests that, from the point of view of the local observer, the coherent wave package in late times corresponds to the classical probability distribution of background metrics.
Thank you for your attention!